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On the Cost of Nescience in Energy Decarbonisation

The application of Value of Information Analysis to Energy System planning problems

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Date: 12th August, 2022

I hereby declare that, except where specifically indicated, the work submitted herein is my own original work.

Signed: Max Langtry *Date:* 12th August, 2022

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This dissertation is submitted for the degree of
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ipsa scientia potestas est

[0] Sir Francis Bacon, *Meditationes Sacrae*, 1597 [1]

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation, excluding front matter, contains not more than 12,000 words[1] including footnotes, captions, and tables, but excluding references and appendices.

Max Langtry
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Abstract

The development and operation of future net zero energy systems is subject to uncertainties from a wide and diverse range of sources. Accounting for these uncertainties in energy system models, and being able to understand and quantify their impact on the behaviour of energy systems in a transparent and justifiable manner, is critically important to enabling the identification of system designs that will perform well in expectation in the uncertain future. However, consensus has yet to be achieved in the literature as to which uncertainty modelling techniques will be able to provide the rigorous uncertainty analysis of large-scale energy system models required to properly support energy system decarbonisation policy making. This work studies the suitability of Value of Information Analysis, a Bayesian Decision Analysis based framework for uncertainty quantification, for the study of the impact of uncertainties in energy systems problems. It is found that the framework has significant potential in this regard, and that further research effort is required to expand its applications in the energy systems field. Potentially novel extensions and genearlisations to the framework are proposed, including a reinterpretation of the Value of Information Analysis framework as a sub-field of Stochastic Optimal Control involving the comparison of the expected performance of different control schemes, which enables the study of general control scheme architectures in a Value of Information context, and a demonstration of the equivalence of Monte Carlo approximations of expected utilities to a statistical form of classical sensitivity analysis. The Stochastic Optimal Control extension is applied to the case of Linear Programming based decision problems, and is shown to be able to provide a variety of insights into the effects of uncertainty on state-of-the-art linear Energy System Optimisation Models, most notably the quantification of the expected performance reduction arising due to the presence of uncertainty.

1. Introduction

Transitioning to net zero carbon energy systems is a critical step in achieving the ambitious carbon emissions reductions targets to which most European nations are committed [2]. This transition will require the integration of large proportions of low-carbon renewable power generation into the supply asset mix of existing energy systems. However, adopting renewable power generation into energy systems poses a number of significant Engineering challenges, which arise from the physical nature of the generation technologies and their dependence on met-ocean conditions. Of key importance is the non-controllable, variable nature of power generation from prominent renewable technologies such as wind and solar, which further exhibit limited predictability outside of near-term forecasts and annual aggregates [3]. This intermittency of energy supply which comes with integrating high proportions of renewable generation introduces additional uncertainty into the operation of energy systems, which must be managed and mitigated to ensure that energy network stability and security of energy supply can be maintained in future net zero energy systems, which cannot rely on dispatchable fossil fuel power generation. Managing the spatio-temporal variability of future energy generation asset portfolios will require the development of additional energy transmission infrastructure, as well as supporting auxiliary energy infrastructure such as energy storage & arbitrage, frequency response, real-time energy telemetry systems, and novel energy management strategies, such as demand-side response, real-time pricing, grid interconnection, and sector-coupling [4–8]. Thus, transitioning to net zero energy generation requires substantial adaptation and development of existing energy system infrastructure to facilitate the integration of high proportions of renewable generation, and so imposes a significant additional cost to the provision of low-carbon energy to the consumer.

1.1 Energy Systems Modelling

Modelling future energy systems allows the nature of their behaviour and operation to be studied and understood, and consequently the identification of low-cost system designs [9–15]. Energy systems modelling can therefore be used to guide infrastructure development policy decisions to minimise the cost of the transition to net zero carbon energy to society, and ensure the resilience of energy infrastructure to the uncertainties imposed by variable renewable generation.

Due to the economic significance of maintaining a secure and cost efficient energy supply system, and more recently its importance in enabling decarbonisation, energy systems modelling has received substantial research attention [16–33]. A central question in the study of energy systems is that of determining minimal cost asset investment strategies to fulfill future energy demand, possibly subject to operational constraints (such as system stability, outage risk or net carbon emissions), which are termed ‘capacity expansion planning problems’. Early models were put forward and discussed in the literature in the 60s & 70s [16–19], however modern advancements in computational capabilities and historic weather measurement data availability [34–41] led to further developments in the field and the expansion of the analysis capabilities of Energy System Models (ESMs). Most significantly, recent contributions to the literature developed high spatio-temporal resolution models of national scale energy systems, for which minimal cost system design solutions could be tractably obtained whilst analysing hourly-resolved operation over durations of the order 10 years [20–22], enabling the accurate long-term planning of future decarbonised energy systems.

Over the history of the literature an extremely large number of models have been proposed, covering a wide range of modelling approaches and mathematical formulations, including Linear Programs [18, 20], Stochastic Programs [42, 43], Mixed Integer Linear Programs [44–46], and general Mixed Integer Non-Linear Programs [47–49]. As discussed in [33], the saturation of models in the literature leads to significant issues for policy makers relating to model selection and the lack of comparability of model results. This work focuses on Linear Programming (LP) based ESMs, such as [20–22], as due to their computational efficiency advantages over other model types and the resulting ability to handle high spatio-temporal resolution models, they are most suitable for the study of long-term asset development planning problems for large-scale future energy systems. Additionally, a recent study demonstrated that linearised models can provide sufficiently accurate representations of non-linear network effects at low computational cost [50].

The operating conditions of future energy systems, which strongly influence their optimal designs, are subject to a large and diverse set of underlying uncertainties, including energy demand, fuel costs, and network availability. Further, when looking towards the design of decarbonised energy systems and the integration of renewable generation technologies the set of underlying uncertainties is expanded significantly, with the addition of uncertainties in regulatory & policy frameworks, CO_2e emissions prices, generation costs from renewable sources, auxiliary technology availability and performance, and temporal power generation from renewable sources, amongst others [51–57].

A large number of methods for quantifying the impact of these underlying uncertainties on the system design solutions identified by ESMs have been proposed in the literature, as well as techniques for accounting for underlying uncertainties within the models, to allow for system designs which perform well under such uncertainty to be identified [31, 58]. However, whilst the literature recognises the importance of addressing uncertainty in ESMs[2] [27, 31], it is yet to reach consensus on which methods adequately capture the impact of the criti-

[2] Reference [27] comments that whilst "[the i]mportance of uncertainty [is] widely acknowledged, . . . only a minority of papers use a formal uncertainty analysis methodology". "Without adequately addressing uncertainties, the model insights may be limited, lack robustness, and may mislead decision makers".

cal underlying uncertainties on the operation and optimal design of energy systems. The development of a rigorous, statistical method for quantifying the impact of uncertainties on system performance, and incorporating it into modelling techniques to achieve performant, statistically informed system designs, remains an open area of research.

1.2 Value of Information Analysis

Value of Information Analysis (VoIA) is a Bayesian method for quantifying the expected value of uncertainty reduction arising from the gathering of data in the context of a defined decision problem [59, 60], which was originally proposed by Raiffa [61] in 1968. It has subsequently found applications across a range of Engineering disciplines, including structural health monitoring in Infrastructure Engineering [59] and seismic surveys in Oil & Gas Engineering [62], as well as in many other fields of study such as Agriculture, Environmental Science, Economics, and Medicine [63, 64]. However, at present there have been a limited number of applications of this Bayesian Decision Analysis technique to energy systems problems.

1.3 Thesis Stratagem

This work seeks to explore the potential applications of VoIA to energy systems problems, and the ways in which the methodology can contribute to the understanding of the behaviour of future, net zero energy systems under uncertain development and operational conditions. Further, it investigates how the VoIA framework can be generalised and extended to allow for the study of more complex systems, and demonstrates the ability of such a generalisation to provide a transparent and justifiable method for quantifying the impact of uncertainties on the performance of Linear Programming based energy system optimisation models.

The presented work is structured as follows. A review of the literatures concerning techniques for accounting for the impact of uncertainties in ESMs, and the application of VoIA to ESMs, including a critique of existing VoIA applications, is performed in Section 2. Section 3 provides an overview of the theory of classical VoIA, before presenting an extension and generalisation of the existing framework. The extended VoIA framework is then applied to Linear Programming based decision problems in Section 4, in which the abilities of this application to provide a range of insights on, and measures of, the impact of uncertainties on the performance of current state-of-the-art LP-based Energy System Optimisation Models (ESOMs) are demonstrated. A numerical example of which is presented in Section 5. Finally, Section 6 summarises the findings and proposes a series of promising research directions for future works.

2. Literature Review

2.1 Energy Systems Modelling under Uncertainty

The goal of energy systems modelling is to gain insight into the nature and behaviour of future energy systems, and from this understanding, determine how to optimally design, adapt, and operate both the networks of physical assets comprising the energy system and the surrounding markets and regulatory structures that allow it to interact with its users, subject to some determined operational objectives and constraints. The performance of system designs and operational strategies, which themselves influence the performance of system designs as the operational capabilities of a design contribute to its overall merit, are dependent on the characteristics of the physical assets implemented and their interactions, the behaviour of interfacing markets & users, and the operational conditions to which the system is subject. All of these aspects contain significant uncertainties, which are further compounded by the predictive nature of the task of designing future systems. As a result, the modelling of future energy systems, and any derived results, are subject to a wide and diverse set of sources of uncertainty. These uncertainties must be addressed and accounted for by ESMs so that decision makers can have confidence that the guidance they provide will lead to future energy systems that perform well under the true observed state of the world when it occurs, i.e. that the system designs are 'performance robust'[3] to the underlying uncertainties.

2.1.i) Sources of Uncertainty in Energy Systems

When considering the application of ESMs to the design of decarbonised energy systems, and the additional challenges and uncertainties introduced by the integration of variable renewable sources into the generation mix, some of the key sources of uncertainty identified in the literature are:

- renewable power generation derived from stochastic weather conditions, both volumes and spatial & temporal patterns [51, 65]
- climatic conditions that drive weather patterns [52, 53]
- cost of renewable generation technologies [54]
- costs & availability of supporting auxiliary infrastructure technology, such as energy storage and transmission capacity [55]
- energy demands, both volumes and spatial & temporal patterns [56, 65]
- consumer & market response behaviours

[3] A system design is said to be 'performance robust' if it provides a 'good' level of performance, in terms of design objective, over the probability space defined by the underlying uncertainties within the system. Its precise mathematical definition is dependent on both the system context and objectives of the decision maker.

- climate & energy policies and associated regulatory systems, such as emissions standards [66, 67]
- cost of carbon emissions [68]
- cost of (carbon intensive) dispatchable power generation [68]

Note that this work considers only the parametric uncertainty associated with ESMs, the uncertainties in the values of the model parameters, and not structural or model uncertainty [33], hence does not assume any particular model structure.

2.1.ii) Methods for Addressing Uncertainties

The literature proposes a variety of different methods for accounting for the impact of these underlying uncertainties on the results produced by ESMs, and for incorporating measures of uncertainty into the system design process to produce solutions that are 'performance robust' to it. These methods can be broadly categorised into two types of approaches based on whether the underlying uncertainties are represented explicitly within the model, or whether the uncertainties are treated externally to the model and their impact studied through the evaluation of the model at test or sample points from the probabilistic input space, which will be called 'explicit representation methods' and 'evaluation based methods' respectively. Explicit representation methods tend to produce a single 'statistically' informed system design solution, and due to the incorporation of probabilistic representations into the model that must be either evaluated or optimised over, tend to have much greater computational costs per evaluation compared to their deterministic equivalents. Evaluation based methods on the other hand are used to determine the output space of system design solutions resulting from the probabilistic space of input parameters for the model being evaluated. As a result these methods are used to produce measures of the impact of underlying uncertainties on either the performance of given system designs, or the characteristics of system design solutions produced by the model being investigated. Due to their external treatment of stochasticity, and resulting simplicity, they can be applied to the study of more complex ESMs, and are fundamentally suited to compute parallelisation.

A literature review identified the application of the following methods for handling uncertainty to ESMs:

- Evaluation based methods
  - scenario analysis & possibilistic methods [9, 30, 58, 69]
  - $\sim$ probabilistic methods (scenario trees, Monte Carlo methods) [27, 31, 58]
  - $\sim$ hybrid possibilistic-probabilistic techniques (mixed scenarios & samples) [58]
  - global sensitivity analysis & interval analysis [27, 30, 31, 58, 70]
  - Methods for Generating Alternatives (MGA) [27, 30, 31, 71–73]

- Explicit representation methods
  - $\sim$ Stochastic Optimisation [27, 30, 31, 42, 43]
  - $\sim$ chance/risk constrained optimisation [74–77]
  - robust optimisation [27, 58]
  - fuzzy optimisation [58]
  - information gap decision theory [58]
  - $\sim$ Value of Information Analysis (VoIA) [42, 43, 66–68, 78–91]

Reviews [27] & [30] provide a thorough overview of many of these techniques.

These methods can be further categorised into statistical methods (denoted with $\sim$ in list above) and non-statistical methods, depending on whether the representation of uncertainty in the model exploits the full statistical distributions of the uncertain parameters, or uses some non-statistical proxy of parametric uncertainty such as a parameter range.

Whilst VoIA is typically a statistical explicit representation method, which also contains a sampling and evaluation type step, it can be generalised for use as a statistical evaluation based method. This extension of standard VoIA is discussed in Section 3.4.ii).

Statistical methods are greatly preferable over non-statistical methods for design analysis, as they provide decision makers with information on not only what outcomes *can* occur, but which outcomes are *likely* to occur, either in terms of expectations or full probability distributions. However, these derived output distributions are predicated on the chosen input parameter distributions. A key criticism of statistical methods, including Bayesian analysis, raised in the literature is that the prior and likelihood distributions for the models must be chosen during the definition of the model. Hence, statistical methods suffer from similar problems of lack of robust justification and opaque 'reasonable choices', to non-statistical methods [92]. Thus it is argued whether statistical methods really improve the quality of information provided in the modelling, or whether they merely increase the complexity of models and their results without improving insights or accuracy. However, in many instances parameter distributions can be learned from data [60, 62, 68, 93], such as historic measurements or simulation results, which addresses such criticisms and should be practiced wherever possible.

2.1.iii) Usage of Methods in Existing Literature

Due to their simplicity and interpretability, scenario analysis and global sensitivity analysis are the most common uncertainty assessment methods applied to ESMs in the literature [9–11, 13, 69, 94]. However recently more advanced techniques have experienced greater research attention, in particular MGA [71–73] and Stochastic Optimisation [27, 30, 31, 42, 43, 95]. Both methods seek to identify system design solutions that are 'performance robust' to uncertainties, but whilst Stochastic Optimisation explicitly models parametric uncertainty in its optimisation formulation and optimises over expected performance, MGA seeks to explore the space of near-optimally performant system designs to identify solutions that are robust to model uncertainties, and so is less relevant to the study of energy system uncertainty in this work.

Of the uncertainty assessment methods identified in the literature review, Stochastic Optimisation provides the most complete statistical representation of the underlying parameter uncertainties in energy system models, though correspondingly has the greatest modelling and computational complexity. It is also found to be the key model type to which VoIA is applied in the literature for this reason. Examples of such applications are discussed in Section 2.2.

The number and diversity of methods for investigating the impact of uncertainties on ESMs applied in the literature demonstrates that a consensus has yet to be reach on which methodologies are able to robustly quantify this impact of uncertainty. Hence, the development of a rigorous analysis methodology which provides a complete statistical treatment of uncertainty remains an open research question. Due to the deficiencies of existing methods, the literature review did not identify any that were capable of addressing the inherent stochasticity of variable renewable generation, and thus the impact of this substantial uncertainty in future low-carbon energy systems is yet to be studied. Understanding this impact is critical to the effective transition to net zero energy systems, and so highly motivates further study in this research area.

2.2 Value of Information Analysis in Energy Systems Modelling

2.2.i) Discussion of Previous Applications

Since the development of both Energy Systems Modelling [18] and Value of Information Analysis [61] in the 50s & 60s respectively, a number of applications of VoIA for the quantification of the impact of underlying uncertainties on ESMs have been proposed in the literature. However, VoIA has not received as much research attention as other uncertainty assessment methods outlined in Section 2.1.ii).

A review of existing literature identified 19 applications of VoIA to energy systems problems. However, due to time limitations on this work this literature review was not extensive, and further literature search is required to fully explore the extent of existing VoIA applications to ESMs. Appendix A of [67] provides a further review of the use of stochastic programming for energy systems analysis, discussing the use of VoIA within the identified studies.

Table 1 presents a summary of the characteristics of the chosen model formulation for each application identified, and the uncertainties studied within the analysis.

Paper		Formulation			Uncertainties considered								
Ref.	Year	Analysis type	# Decision stages	RV Dist. type	Energy demand	Renewable generation	Technology performance	Technology availability	Energy/tech. costs	Supporting inf. costs	Carbon cost	Climate policy	Economic factors
[78]	1987	CP	2	D	•								
[79]	1990	CP	2	D	•				•				
[80]	1990	CP	4	D	•				•				
[81]	1998	CP	3	D	•							•	
[42]	2007	CP	2	D	•	•					•		
[82]	2008	EC	1	N/A									•
[66]	2009	CP	N	CT							•		
[83]	2009	EC	1	D								•	•
[84]	2013	CP	5	D	•	•							
[85]	2013	CP	2	D				•				•	
[86]	2014	CP	2	D				•				•	
[67]	2015	CP	2	D			•	•	•	•		•	
[87]	2017	OP	N	CT		•							
[88]	2018	CP	2	D	•								
[68]	2019	CP	1	C			•			•	•		
[43]	2019	CP	2	DT	•								
[89]	2019	CP	1	D								•	
[90]	2019	CP	2	D		•							
[91]	2021	OP	1	C		•							

Analysis types: capacity expansion planning (CP), system operation (OP), economic modelling (EC); N Decision stages indicates models which have been formulated as multi-stage stochastic programs of arbitrary length (no. of stages); Random Variable (RV) distribution types: discete (D), continuous (C), the addition of the symbol (T) indicates that the modelled random variables are timeseries vectors.

Table 1: Summary of model characteristics and considered uncertainties in VoIA applications to Energy Systems Modelling identified in literature review

Of the studies identified, most were found to formulate their decision problems using classical decision tree based stochastic programs. Further, most of the formulated decision problems were structured as two-stage[4]stochastic programs, with an initial planning decision step followed by an operational decision step. A select number of studies chose to use either simplistic single-stage stochastic programs [68, 82, 83, 89, 91], or more complex multi-stage problem structures [66, 80, 81, 84, 87].

Reference [66] presents a different approach to multi-stage stochastic decision problems, applying Bellman optimality [96] to determine the expected utility maximising decision path, whilst [87] employs an optimal forwards scheduling algorithm which approximates operation of a hydro-electric plant as a Markovian system and uses a Least Squares Monte Carlo approximation to compute the expected utilities of the control decisions. In [79] an alternative technique for describing the structure of energy system decision models and performing stochastic utility computations in proposed, that of influence diagrams representations, which provide greater clarity and interpretability of the structure of the decision problem and dependencies within the underlying uncertainties. Finally, reference [82] presents a game theoretic model of investment in electricity markets, which is comprised of a one-shot game, and considers uncertainty in the shared information available to firms on the parameters defining the game structure.

2.2.ii) Literature Critique

Statistical Representation of Parametric Uncertainties

The statistical treatment of parametric uncertainty in existing applications in the literature demonstrates some significant limitations, with data-driven statistical methods such as sampling-based Monte Carlo methods with learned parameter distributions being used infrequently. As demonstrated by Table 1, most applications opt to use discrete approximations of parameter distributions, neglecting the continuous nature of many uncertainties, with many employing simplistic scenario based methods to which probabilities are assigned and justified by ‘expert knowledge’ and which outcomes the authors deem ‘likely’. Such methods show only limited advancements on traditional scenario based modelling, as they rely heavily on qualitative judgements in their formulation. Whilst statistical information is introduced into the modelling, its accuracy either has not or cannot be verified, and hence these methods suffer from issues of lack of transparency and justifiability. [68] employs Monte Carlo methods to continuously distributed uncertain parameters, however, whilst doing so it considers only a one-stage stochastic problem and severely limits the available action space to only 15 system design strategies. Hence, even the existing methods which use advanced statistical techniques exhibit other weaknesses in the quality of their statistical treatment.

[4] The number of ‘stages’ in a stochastic program refers to the number of iterations of successive decisions and revelations of the state of the world that occur over the course of the defined decision problem, rather than the number of time periods in the model within which decisions are made.

Therefore, significant research effort is required to improve the completeness of the statistical analysis applied to ESMs through the VoIA framework whilst maintaining computational tractability[5]. Of particular importance in this regard is the widespread adoption of data-driven uncertainty models, where parametric distributions are learnt from experimentation and historic data [60], as this provides techniques for selecting probability distributions that are examinable and justifiable, removing qualitative judgements from the decision analysis. Some critiques remain over the validity of even data-derived distributions, as certain characteristics such as the significance of parameter covariance and the challenges associated with quantify it, and the evolution of underlying distributions due to changes in the system state, such as those induced by climate change [57], raise questions about whether the true underlying distributions can be known or accurately modelled.

Studied Uncertainties & Analysis Comparability

The VoIA applications identified in the literature considered a broad range of sources of uncertainty in their analyses. The most common uncertainties considered were those relating to future energy demands, costs of both fuels and technologies, and climate policies. However, as can be seen in Table 1, few studies managed to analyse the effects of a significant proportion of the set of underlying uncertainties simultaneously. This raises questions about the validity of the results derived from these studies, as the impacts of uncertainties on a given energy system decision problem are highly inter-dependent [62, 68]. Thus, when quantifying the impact of a given uncertainty, and identifying the critical uncertainties for a particular decision problem, these results are only valid in a very restricted context, i.e. for that particular decision problem and model structure where only the considered uncertainties are acting. As all underlying uncertainties impact true energy systems, this context may not be representative of the physical system being studied, and hence the results may not be valid. Analyses with such limited considerations of uncertainty may lead to erroneous results if the underlying causal relationships between uncertainties are not represented in the problem through the omission of a critical uncertain variable.

Further issues of result comparability arise, as of the studies identified in the literature, very few study the same ESM. This is partly due to the saturation of ESMs in the literature [25, 33]. As each ESM contains unique subtitles within its formulation, each model responds differently over its (potentially unique) input space, and hence the impacts of given uncertainties may differ from model to model. Any discrepancies between results derived from the uncertainty study of different models are highly challenging if not impossible to robustly reconcile, as a qualitative judgement must be made as to which model(s) most accurately

[5] When applying sampling-based Monte Carlo methods to Bayesian Decision Analysis problems formulated using classical scenario trees, the computational cost of model evaluation is at least multiplicative in the number of samples drawn from the underlying distributions of uncertain parameters, i.e. $\mathcal{O}\left(\prod_i n_i\right)$, where n_i is the number of samples drawn for parameter i

represent the true physical system, which in many cases is hypothetical and non-testable. Further, this raises doubts about whether obtained results generalise across models. Demonstrating this by evaluating an ensemble of models and observing result consensus presents a significant increase in the computational and research time cost of performing uncertainty analyses. The literature review did not identify any studies which compared the results of VoIA applications between model formulations.

The uncertainty introduced by the severely limitedly predictable variability of many renewable generation technologies is of critical importance to the design of future net zero energy systems [97], and hence the study of its impact is a key area for energy systems research. Whilst some studies identified in the literature sought to account for this renewables variability related uncertainty, many did so via crude approximations of the true, highly complex nature of renewable generation, which contains many relatively poorly understood spatio-temporal characteristics and internal causalities. Approaches commonly taken were to either both spatially and temporally aggregate the analysis, removing much of the complexity of the uncertainty at the expense of model accuracy, or to neglect timeseries dependencies whilst retaining temporal resolution, considering only uncertainty in capacity factors which are then used to augment the measured data. However neither of these methods are able to quantify the true impact of renewable generation uncertainty on energy system design, which is of significant importance to net zero policy development. Therefore, the development of statistical analysis methods that are capable of handling and quantifying the impact of this complex uncertainty associated with renewable generation is a research area which must receive substantial attention for the field to progress.

Studied Model Scopes & Problem Types

Further, existing studies have demonstrated very limited capabilities in accounting for and addressing the complex behaviours of large-scale energy systems, with most works employing extensive simplifications in their model formulations. Such simplifications include removing spatial behaviours by aggregating energy flows to the national level, and removing temporal behaviours by aggregating to annualised operation [42]. These aggregations are employed to make the model formulations and their results more accessible and interpretable to human decision makers, and to reduce the computational cost of the ESMs such that existing uncertainty analysis methods can be applied in a tractable computation time. However, the literature demonstrates that such aggregations which limit a model's ability to represent temporal variation in renewable generation and energy transport effects have a significant impact on the accuracy of the derived results [98–101]. Uncertainty quantification methods of sufficient computational efficiency must be developed so that the impact of energy system uncertainties on large-scale, high spatio-temporal resolution ESMs such as [20–22] can be studied, to provide accurate and reliable analyses to inform policy & decision makers. The generalisations of the VoIA framework discussed in Sections 3.4.i) & 3.4.ii), which are partially implied by the model formulations and discussions presented in [60] & [87], may provide a pathway to developing such improved methods.

Whilst the study of capacity expansion planning problems is of great importance to the cost-effective decarbonisation of modern energy systems, many other pertinent problems exist in the field of energy systems modelling which could be studied using the VoIA framework, and whose solution could make contributions to decarbonisation efforts. However, the existing literature focuses primarily on capacity expansion planning problems, and the literature review identified few other types of energy system problems to which VoIA has been applied. A series of alternate problems to which VoIA could be applied are proposed in Appendix A.

Applied Value of Information Metrics

Of the VoIA applications to ESMs identified in the existing literature, no studies considered the case of imperfect information collection [59] and the associated Expected Value of Imperfect Information (EVII), with only the more conceptually and computationally simple metrics of Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS) being evaluated. This is likely primarily due to the problems and uncertainties being studied in these applications, as considerations of imperfect information are most applicable to uncertainties for which there exists an option for physical measurement, to which VoIA can be applied to evaluate the net benefit of measurement. The literature studies mostly future energy demand, cost, and climate policy uncertainties, which do not have the option of ex-ante measurement. However, EVII can be applied to analyse the impacts of R&D investment and improved forecasting [102], and so imperfect information studies could contribute additional understanding to the literature in future.

3. Theory of Value of Information Analysis

A brief overview of the Bayesian Decision Analysis (BDA) formulation and theory of the classical Value of Information Analysis (VoIA) framework is presented in Sections 3.1 to 3.3, before generalisations of and extensions to the framework are discussed in Section 3.4. References [42, 59, 60, 67] provide further detail on the theory underlying VoIA.

3.1 Bayesian Decision Analysis

3.1.i) Stochastic Decision Analysis

In Stochastic Decision Analysis (SDA), the aim is to determine the optimal action which maximises the expected utility obtained from a system which is subject to some underlying uncertainty.

The generalised one-stage problem is defined as the selection of an action from the available action space, $a \in \mathcal{A}$, in the presence of some uncertain state of the world, $\theta \sim \pi(\theta)$, which acts on the system and leads to some returned utility[6], $u(a, \theta)$, once the uncertain state of the world is revealed. Hence, the action a must be taken before the uncertain parameter θ can be observed. These problems can be extended to contain multiple decision stages, where a successive sequence of n actions and revelations of uncertainty occur.

These decision problems are often described using decision trees[7], which visualise the evolution of the system arising from the different possible action pathways and the outcomes that result [104], and provide an easily interpretable description of the problem that allows expert knowledge to be readily incorporated into the description of complex systems. Figure 1 shows the decision tree for the general one-stage Stochastic Decision Problem (SDP).

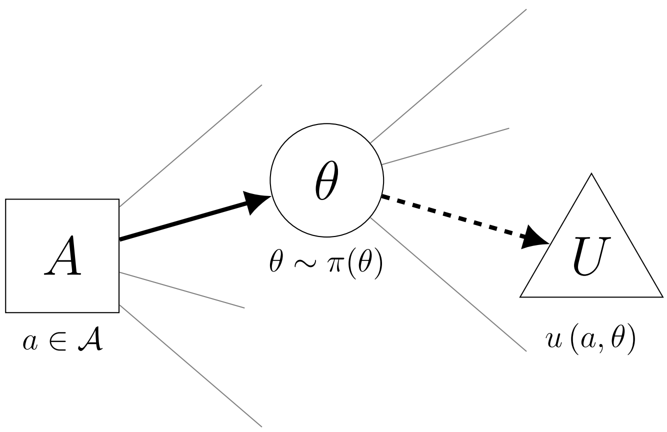


Figure 1: Decision tree representation of Prior Decision Problem

Adapted from Figure 1 of [59]

The process for solving the SDA can be formulated as a Stochastic Optimisation,

$$\max_{a \in \mathcal{A}} \mathbb{E}_{\theta} \left\{ u(a, \theta) \right\} \tag{1}$$

where the solution to the optimisation, a^*, is the determined optimal action for the decision problem. The determination of a^* requires the decision maker to know the true prior distribution over the uncertain state of the world $\pi(\theta)$.

[6] Costs are defined as negative utilities

[7] This Directed Acyclic Graph representation of Stochastic Decision Problems enables them to be readily analysed using Bayesian Networks [103]

This formulation is fully general and admits action vectors in arbitrary spaces, which are restricted to be members of a general constraint set $\mathcal{A}$. The problem definition can be extended to allow the distribution of θ to be dependent on the action taken, a, i.e. $\theta \sim \pi(a, \theta)$. This corresponds to the case of remedial actions which influence the state of the system.

In the context of BDA, this problem is known as the 'Prior Decision Problem', as the decision maker has only information on the prior distribution of the uncertain parameter θ.

3.1.ii) Deterministic Decision Analysis

Whilst the solution a^* is stochastically optimal, alternative weakly sub-optimal solutions can be identified by means of equivalently formulated deterministic methods at lower computational cost. The most commonly used deterministic method is the deterministic optimisation derived when stochasticity is remove by assuming the uncertain parameter θ to take on it's expected value, $\bar{\theta} = \mathbb{E}\{\theta\}$, the solution to which is denoted $\hat{a}$. This deterministic approximating optimisation is termed the Expected Value Problem (EVP) [42, 43, 89].

3.1.iii) Bayesian Decision Analysis

The SDA is brought into a Bayesian setting through the introduction of a second, preceding decision stage in which the decision maker is provided with a set of measurements options E. By taking a measurement action $e \in E$, which results in measured data $z \sim \pi(z)$, the decision maker can construct the resulting posterior over the uncertain parameter θ, $\theta \sim \pi(\theta|z) \propto \pi(\theta) f(z|\theta)$, where $f(z|\theta)$ is the measurement model[8]of the system. This posterior distribution, and the improved information it provides on θ, can then be used to inform the second stage action, a, the decision maker takes, improving its performance. Using the prior distribution over data measurements, $\pi(z)$, the stochastically optimal measurement-action pathway can be determined.

The process for solving the BDA can be formulated as a two-stage Stochastic Optimisation,

$$\max_{e \in E} \mathbb{E}_z \left\{ \max_{a \in \mathcal{A}} \mathbb{E}_{\theta|z} \left\{ u(a, \theta) \right\} \right\} \tag{2}$$

the solution to this optimisation comprises an optimal measurement action, e^{**}, and an optimal action function which is dependent on the measured data, $a^{**}(z)$, i.e. the expected

[8] For problems in which there exists the option to make measurements of different quantities, $z \to \phi \in \Phi$, that provide different information on the uncertain state of the world, θ, then the probability model of each measured quantity, $f_\phi(\phi|\theta)$, must be used in the computation of the resulting expected utility. This complexity is neglected for the sake of notational simplicity. For further detail see [59]

utility maximising action for the given posterior distribution, $a^{**}(z) = \max_a \mathbb{E}_{\theta|z}\left\{u(a,\theta)\right\}$.

In the context of BDA this problem is known as the 'Pre-Posterior Decision Problem', as the initial measurement decision is made before any data is measured and hence any posterior distribution can be constructed, but is made with the knowledge that the posterior distribution can be exploited in the second stage decision. The decision tree representation of this Bayesian Decision Problem (BDP) is given in Figure 2.

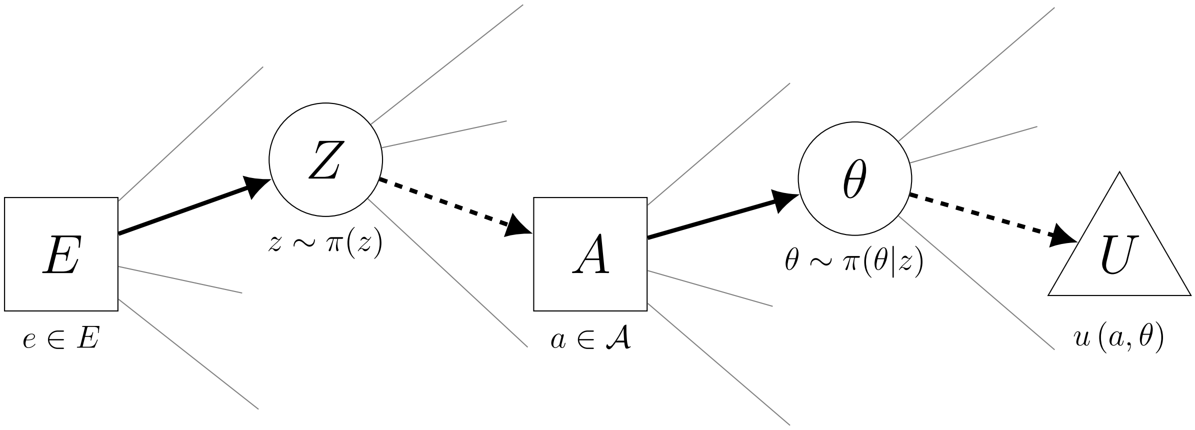


Figure 2: Decision tree representation of Pre-Posterior Decision Problem

Adapted from Figure 1 of [59]

The BDA framework requires the decision maker to have knowledge of both the true data likelihood and conditional probability distributions, $\pi(z)$ and $\pi(\theta|z)$. Therefore, the definition of a valid BDP requires the construction of a valid underlying joint probability distribution for the system, $\pi(\theta, z)$, which describes the distributions of the uncertain state of the world and the measurement model of the system, $f(z|\theta)$, and leads to the required marginal and conditional distributions.

The presented model is general, admitting analysis of both perfect and imperfect measurement information. For the case of perfect information, the uncertainty in the state of the world, θ, collapses when the measurement z is made, and the conditional probability distribution collapses to a Dirac delta, $\pi(\theta|z) \to \delta(\theta - z)$, making the expectation over $\theta|z$ deterministic. The model can also be extended to allow for the introduction of uncertain measurements, i.e. when a raw measurement z' is made, the true value of the quantity being measured is distributed as $p(z|z')$. Hence the BDA optimisation becomes,

$$\max_{e \in E} \mathbb{E}_{z'}\left\{\mathbb{E}_{z|z'}\left\{\max_{a \in \mathcal{A}} \mathbb{E}_{\theta|z}\left\{u(a,\theta)\right\}\right\}\right\} \tag{3}$$

In the Bayesian framework, it can be seen that the expected utility achieved by the Prior Decision Process implicitly takes expectations over data measurements,

$$\mathbb{E}_{\theta,z}\left\{u(a^*,\theta)\right\} = \mathbb{E}_z\left\{\mathbb{E}_{\theta|z}\left\{u(a^*,\theta)\right\}\right\} = \mathbb{E}_\theta\left\{u(a^*,\theta)\right\} \tag{4}$$

as the action taken is not dependent on any measurement data as the decision maker does not have access to such information, $a^*(z) = a^*$. This follows from the expansion of the integral form of the expectations.

3.1.iv) Expected Utilities of Decision Processes

In the formulation of the BDA problem it has been shown that the decision making task under uncertainty can be performed using four different processes, which respectively involve the solution of the following decision problems: Expected Value Problem (EVP), Prior Decision Problem, Perfect Information Pre-Posterior Decision Problem, Imperfect Information Pre-Posterior Decision Problem.

A fifth decision process, involving the solution of the Perfect Control Decision Problem and which is related to the Value of Control, is proposed in the literature [85]. In this case, the state of the world θ is assumed controllable, and the utility maximising state-action pair is sought. However, in this instance of controllable θ statistical analysis becomes redundant as the problem collapses to determinism, hence this decision process is not discussed further as it is not relevant to the study of methods for quantifying the impact of uncertainty considered in this work.

Table 2 summarises the five decision process that can be applied to the BDP, and importantly provides the expected utilities that each of them obtain.

For more complex decision problems, for instance those with continuous uncertain parameters θ for which decision trees cannot be constructed, it may not be possible to determine the expected utilities exactly. In these cases the required expectations are approximated using Monte Carlo methods, with samples from the appropriate underlying distributions drawn using some appropriate method such as Markov Chain Monte Carlo techniques. Some expected utility evaluations, such as the case of uncertain measurements (Equation (3)), require multiple layers of expectations to be computed. The computational implications of this are discussed in Appendix B.

$$\mathbb{E}_\theta\left\{u(a,\theta)\right\} \approx \frac{1}{N}\sum_{j=1}^{N} u\big(a,\theta^{(j)}\big) \quad \text{where } \theta^{(j)} \sim \pi(\theta) \quad \forall\, j \tag{5}$$

Decision Process	Optimal Action Pathway		Expected Utility	
Expected Value Problem	$\hat{a}$	$\underset{a \in \mathcal{A}}{\operatorname{argmax}}\, u(a, \bar{\theta})$	$\hat{y}$	$\mathbb{E}_{\theta}\left\{u(\hat{a}, \theta)\right\}$
Prior	a^*	$\underset{a \in \mathcal{A}}{\operatorname{argmax}}\ \mathbb{E}_{\theta}\left\{u(a, \theta)\right\}$	y^*	$\mathbb{E}_{\theta}\left\{u(a^*, \theta)\right\}$
Imperfect Information Pre-Posterior	$e^{**}, a^{**}(z)$	$\arg\ \max_{e \in E}\ \mathbb{E}_z\left\{\max_{a \in \mathcal{A}}\ \mathbb{E}_{\theta \mid z}\left\{u(a, \theta)\right\}\right\}$	y^{**}	$\mathbb{E}_z\left\{\max_{a \in \mathcal{A}}\ \mathbb{E}_{\theta \mid z}\left\{u(a, \theta)\right\}\right\}$
Perfect Information Pre-Posterior	$e_p^{**}, a_p^{**}(z)$	$\arg\ \max_{e \in E}\ \mathbb{E}_z\left\{\max_{a \in \mathcal{A}}\ u(a, z)\right\}$	y_p^{**}	$\mathbb{E}_z\left\{\max_{a \in \mathcal{A}}\ u(a, z)\right\}$
Perfect Control	$\theta^{\max}, a^{\max}$	$\underset{\theta \in \Theta,\, a \in \mathcal{A}}{\operatorname{argmax}}\ u(a, \theta)$	$y^{\max}$	$\max_{\theta \in \Theta,\, a \in \mathcal{A}}\ u(a, \theta)$

Table 2: Performance of decision processes

3.2 Value of Information

The Value of Information (VoI) is defined as the increase in the expected utility obtained from the solution of a decision problem when a decision maker is provided with additional information relating to said problem, typically concerning the uncertain state of the system.

For a risk neutral decision maker, VoI is always non-negative [43]. Proof of this is provided in Appendix C.

As demonstrated in Section 3.1, a range of different decision processes which incorporate varying degrees of information on the decision problem can be applied to a general BDP. Therefore, a set of different VoI metrics can be defined. This Section presents the three most common VoI metrics applied in the literature: Value of Stochastic Solution (VSS) [67, 84, 90], Expected Value of Perfect Information (EVPI) [60, 67, 84], Expected Value of Imperfect Information (EVII); and discusses their interpretations.

3.2.i) Value of Stochastic Solution (VSS)

The VSS is defined as the difference between the expected utilities achieved by the EVP and Prior Decision Problem,

$$\begin{aligned} \text{VSS} &= \mathbb{E}\{\text{prior decision utility}\} - \mathbb{E}\{\text{EVP decision utility}\} \\ &= \max_{a\in\mathcal{A}} \mathbb{E}_\theta \left\{u(a,\theta)\right\} - \max_{a\in\mathcal{A}} u\left(a, \mathbb{E}\{\theta\}\right) \end{aligned} \tag{6}$$

It is interpreted as the expected benefit of introducing stochastic information into the decision making process, moving from a deterministic optimisation to a stochastic version that accounts for the uncertainty in outcomes. This expected utility improvement comes at the expense of increased solution complexity for the optimisation.

Large values of VSS in a given domain of decision problems motivate the development of statistically informed decision processes for such applications. The study of VSS will be highly pertinent throughout the development of Digital Twins for automated asset decision making, as it will allow researchers to quantify the impact of underlying uncertainties on the expected utility performance of such systems, and so whether the development of uncertainty informed decision strategies within the Digital Twin ecosystem will provide substantial benefits to the end-users.

3.2.ii) Expected Value of Perfect Information (EVPI)

The EVPI is defined as the difference between the expected utilities achieved by the Perfect Information Pre-Posterior Decision Problem and the Prior Decision Problem,

$$\begin{aligned}\text{EVPI} &= \mathbb{E}\{\text{perfect information decision utility}\} - \mathbb{E}\{\text{prior decision utility}\} \\ &= \mathbb{E}_z \left\{ \max_{a\in\mathcal{A}} u(a,z) \right\} - \max_{a\in\mathcal{A}} \mathbb{E}_\theta \left\{ u(a,\theta) \right\}\end{aligned} \tag{7}$$

Expected Net Benefit of Measurement

The EVPI is traditionally interpreted as the expected benefit arising from measuring the true state of the world prior to making a decision. It can also be considered as a decision maker's willingness to pay for such a perfect measurement. Physical measurements commonly have some associated cost, $c(e)$, hence the expected net benefit of taking a measurement e to support a given decision problem is,

$$\mathbb{E}\{\text{net measurement benefit}\} = \text{VoI}(e) - c(e) \tag{8}$$

If this quantity is positive then it is worth the decision maker investing is the measurement. In this way EVPI can be used to support business decisions by providing a quantitative framework for determining whether investments in costly measurements, such as asset monitoring or the contracting of expert advice, provide net benefit to the business.

Uncertainty Quantification Metric - Cost of Uncertainty

However, the EVPI can also be viewed in the opposite sense as the Penalty of Uncertainty. In this interpretation, the EVPI is seen as the reduction in the expected utility obtained from the decision problem as a result of the state of the world, θ, being uncertain or unknown. In this way it can be used as an Uncertainty Quantification (UQ) metric for the impact that the underlying uncertainty of the problem has on the performance of the decision being made. If the EVPI is small, then the decision is found to be 'performance robust' to the underlying uncertainties. However, if the EVPI is large then the decision is found to be sensitive to the uncertainties. A key strength of EVPI as a UQ metric is that it quantifies the impact of uncertainty both in terms of the model outputs, capturing the propagation of the input uncertainties through the model in an end-to-end fashion, and in units of utility, allowing the impact to be expressed in terms of the objective of the decision maker, which provides a clear and comparable measure of uncertainty cost.

3.2.iii) Expected Value of Imperfect Information (EVII)

The EVII is defined in an analogous manner to the EVPI, however for the case where the measurement z provides uncertain information on θ, i.e. $\theta \sim \pi(\theta|z) \neq \delta(\theta - z)$. Hence it is given by the difference between the expected utilities achieved by the Imperfect Information Pre-Posterior Decision Problem and the Prior Decision Problem,

$$\begin{aligned} \text{EVPI} &= \mathbb{E}\{\text{imperfect information decision utility}\} - \mathbb{E}\{\text{prior decision utility}\} \\ &= \mathbb{E}_z \left\{ \max_{a \in \mathcal{A}} \mathbb{E}_{\theta|z} \left\{ u(a,\theta) \right\} \right\} - \max_{a \in \mathcal{A}} \mathbb{E}_\theta \left\{ u(a,\theta) \right\} \end{aligned} \tag{9}$$

The EVII is upper bounded by the EVPI, with equality achieved in the limit as the information uncertainty tends to zero. EVII characteristic curves, such as that presented in Figure 3, can be used as a decision support tool, plotting the EVII over the range of a parameter quantifying the measurement uncertainty against the cost of measurements of those precisions, to identify regions in which uncertain measurements are economical.

The EVII therefore has the same expected benefit of measurement interpretation as the EVPI, but is less readily applicable in the Penalty of Uncertainty sense.

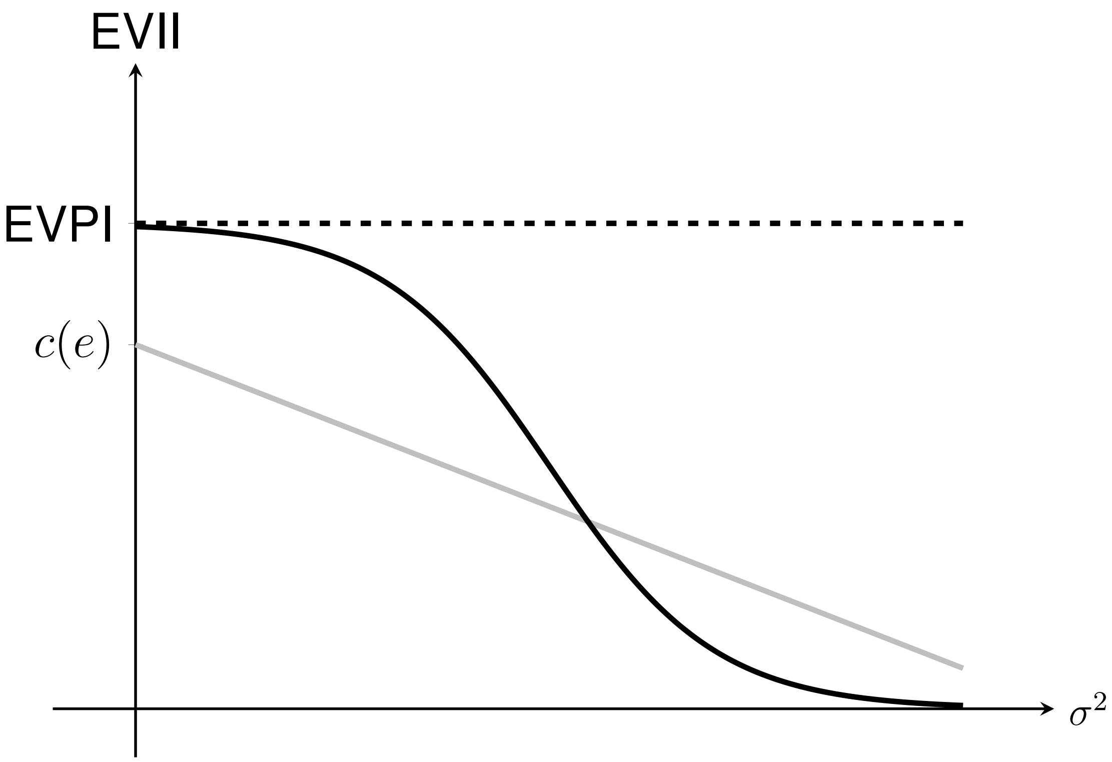


Figure 3: Illustrative EVII vs. measurement cost curve

3.2.iv) Comparison of Value of Information Metrics

Figure 4 presents a visual comparison of the VoI metrics presented and how they relate to the different decision processes discussed in Section 3.1. The figure clearly demonstrates the opportunity for defining additional VoI metrics.

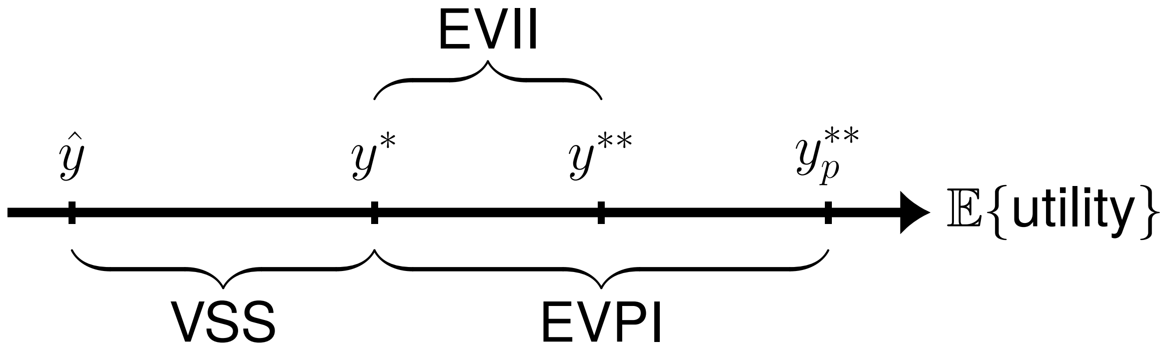


Figure 4: Visual comparison of VoI metrics

Adapted from Figure 1 of [67]

3.3 Critique of Value of Information Framework

As a result of the way the VoIA framework is formulated and the assumptions it makes, there exist a number of weaknesses of the method which must be understood to properly contextualise and interpret the results it produces.

The most significant criticism raised relates to the assumption that the decision maker has perfect knowledge of the underlying statistical distributions of the system, i.e. $\pi(\theta, z)$. In most applications the distributions defining the system are chosen during the model design, however this leads to the standard issues of lack of transparency and justifiability discussed in Section 2.2.ii). Wherever possible the distributions of uncertain parameters should be learnt from data, however even in this case, concerns remain over whether historic or simulation data provides an accurate representation of future system behaviour, and how data sources are selected. If the model distributions chosen do not match the true system then the results of the VoIA are invalid. Though, the significance of this effect can be analysed using the VoIA framework itself, as discussed in Section 3.4.i).

The choice of model distributions has a substantial impact on the results of the VoIA. This means that the computed VoI metric values are only valid within the context of the particular distributions chosen, further compounding the result comparability issues arising from ESM formulation differences discussed in Section 2.2.ii). One option for addressing these concerns surrounding result generalisation is to attempt to demonstrate robustness of the VoIA result to the assumptions made in the model formulation, by performing sensitivity analysis over the model setup, for instance distribution hyper-parameters. However, this incurs a significant additional computational cost multiplication on an already computationally expensive statistical method.

A further critique of the VoIA framework is that whilst the computation of VoI metrics incurs a significant additional computational cost compared to deterministic analysis, explored further in Appendix B, VoIA is only capable of providing decision makers with information on the expected performance of their decision processes. This limits the insight that can be gained from such analysis, as the lack of distributional information prevents uncertainty related risk from being studied through this method.

3.4 Extensions of Value of Information Framework

In this Section a series of extensions to the classical VoIA framework are proposed. Discussions of these extensions were not identified through the literature review, however that literature review was not extensive, and so further study is required to determine the novelty of these approaches in their application to ESMs.

3.4.i) Stochastic Optimal Control Generalisation

Classical VoIA is formulated in the context of a BDP. Within BDA it is assumed that the decision maker is perfectly rational and behaves optimally in terms of expected utility obtained with the information available to them. VoI metrics then compare the Expected Utility Performance (EUP) of decision processes which have access to different sets of information about the BDP. For example, the VSS compares the EUP of the case where only mean estimates of the uncertain parameters are available to that where the prior distributions over the parameters are available, and EVPI compares the EUP of that latter case to the case where the true parameter values are known.

Taking one step into abstraction, the BDP can be considered as a Stochastic Optimal Control problem, where the action decision made by the decision maker is the control scheme implemented to solve the control task, and the formulated decision problem defines the system in which that control scheme acts. In BDA it is assumed that this control scheme is always expectation optimal subject to the quantity of information on the system available, however this assumption can be readily relaxed. This enables the VoIA framework to be generalised to allow for the study of the impact of uncertainty on the EUP of arbitrary control schemes applied to a decision problem. In this work the setup of an arbitrary control algorithm used to determine the taken action for a given decision problem formulation will be termed a Decision System (DS).

Considering a general control scheme $a(\cdot)$, which has access to some information[9] z on the underlying uncertainty of the system, θ, the EUP of this control scheme is given by,

$$y\left(a(\cdot)\right) = \mathbb{E}_{\theta,z}\left\{u\left(a(\cdot),\theta\right)\right\} = \mathbb{E}_{z}\left\{\mathbb{E}_{\theta|z}\left\{u\left(a(\cdot),\theta\right)\right\}\right\} \tag{10}$$

The EUP metric of 'goodness' of control is comparable between control schemes in the context of the chosen DS, and the difference in EUP between control schemes provides a generalised concept of VoI metrics. In this way, it can be seen that this generalisation of the VoIA framework is performing a very simple analysis, the comparison of the EUP of (potentially stochastic) control schemes for a given decision based stochastic control task defined by the DS. Hence, Expected Utility Performance Difference (EUPD) metrics can be defined to study features beyond information availability, and so act as a generalisation of VoI metrics.

In this generalised framework care must be taken when interpreting EUPD metrics, as they are dependent on the controller architecture, and explicit definition of the purpose, valid scope, and interpretation of each metric is required. For instance, an EUPD metric which compares two different controller architectures which have different information availabilities contains contributions from both VoI and controller design performance that cannot immediately be separated.

As with the classical VoIA framework, the EUP of the Perfect Information Pre-Posterior decision process provides an upper bound on the achievable control performance of the DS, and if calculable, can be used as a benchmark for the performance of all other control schemes. The generalised framework is focused more on the direct comparison of EUP values than the more restrictive definition of EUPD metrics. These comparisons must be carefully designed to ensured that they provide the most appropriate information for the desired study, and that the valid contexts of the EUP values are properly accounted for so that the comparisons provide the required interpretation.

Nonetheless, EUPD metrics can be defined to assess the impact that uncertainties have on the performance of control schemes in the context of the uncertain DS. Further, as different controller architectures are comparable under this generalised framework, the EVPI becomes of greater use for systems with non-measurable uncertain parameters, as the EVPI for a given controller architecture provides a UQ metric that quantifies the impact that the underlying uncertainties have on the EUP of that controller. This EVPI UQ metric can be used alongside the base EUP of the controller to provide the meta decision maker with information on both the performance and 'performance robustness' to uncertainty of each control scheme, to better inform controller selection.

[9] The concept of the information z is extended to include distributional information on the uncertain parameters of the DS the control scheme has access to. This information can be zero-dimensional so that non-Bayesian & non-statistical control schemes can be incorporated into the general framework.

Study of High Complexity Systems

A key advantage of the generalised framework is that it allows for the study of simpler control schemes than the stochastically optimal scheme required by BDA. As a result, this allows for the study of far more complex Decision Systems for which the stochastically optimal control scheme cannot be tractably determined, and the analysis of the impact that the underlying uncertainty of the system has on the performance of the decision making process in these highly complex environments. A good example of such complex DSs are those containing large numbers of decision stages, for which the outcome space grows combinatorially, making them rapidly intractable for classical VoIA. Of particular interest is the study of approximations to the stochastically optimal control scheme and their relative performance, which determines their suitability for use as low computational cost controllers. This suggests a fundamental suitability of Reinforcement Learning based methods to generalised VoIA studies, as these advanced control techniques seek to approximate the stochastically optimal scheme. This link to Reinforcement Learning approaches could enable the study of vastly more complex and potentially blackbox systems for which reasonable controllers cannot be analytically designed. This avenue of development requires further research, and may find motivating applications in the uncertainty analysis of Digital Twin based automated asset decision processes. Other potential controller architectures of interest include Markov Decision Processes, which are discussed in a VoI context in [60], and Linear Quadratic Regulators [105].

Section 4 applies the proposed generalised VoIA framework to quantify the impacts of uncertainty in Linear Programming (LP) based decision problems, before a numerical study of an example energy system application is conducted in Section 5.

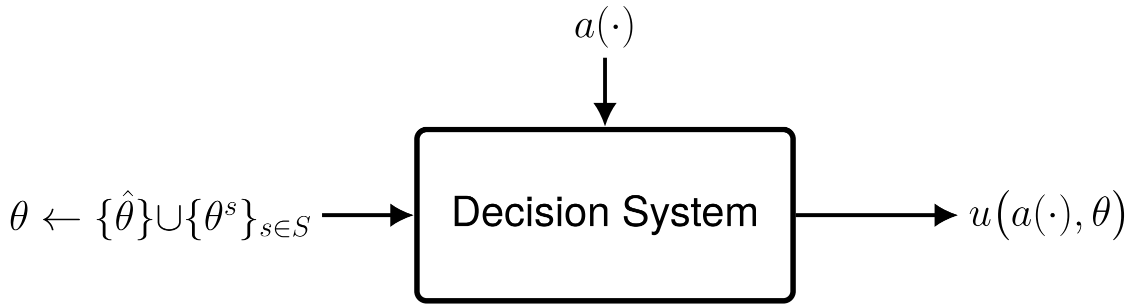
Analysis of Incorrect Priors & Likelihoods and Erroneous Measurements

This extension of the VoIA framework allows it to partially address one of its greatest criticisms, that of the validity of chosen model distributions discussed in Section 3.3, by allowing for the quantification of the impact of incorrect model distributions and measurement errors on the analysis results. The formulation of this type of study is presented in Appendix D.

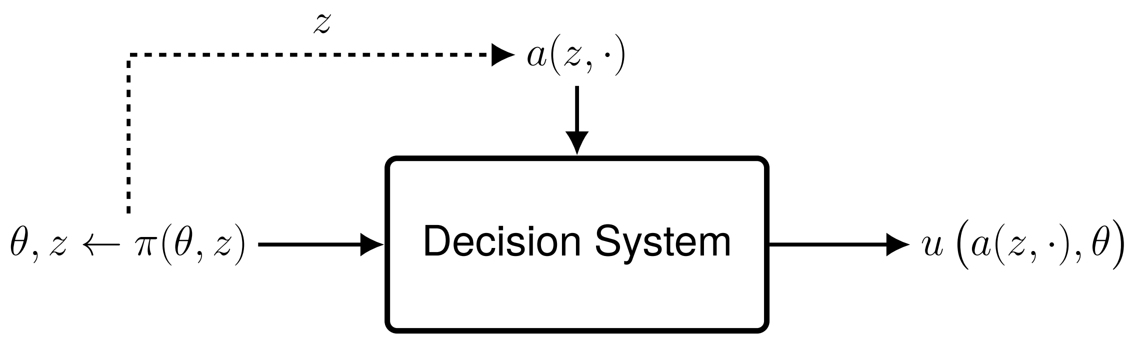
3.4.ii) Generalised Model & Correspondence to Sensitivity Analysis

When considering the case in which Monte Carlo approximation is used to compute the EUP of a control scheme, the procedure of repeated sampling and system evaluation required for the expectation computation can be shown to be analogous to a statistical form of traditional Sensitivity Analysis (SA).

Figure 5 compares stylised representations of the evaluation of a scenario from a SA with the evaluation of a sample from an expected utility computation, for a general DS to which an arbitrary control scheme is applied. In SA, the model, in this case the application of a given control scheme to the DS, is evaluated over a set of parameter value scenarios, $\{\theta^s\}_{s\in\mathcal{S}}$, selected by an expert user, and the resulting utility performance is compared to that of the original case, $\hat{\theta}$. When the EUP of a control scheme is evaluated via Monte Carlo approximation, a set of samples are drawn from the underlying parameter distribution, $\{\theta^{(j)} : \theta^{(j)} \sim \pi(\theta)\}_{j=1}^{N}$, and the model utility performance is then evaluated for each sample, before the mean utility performance is computed and used as the expectation estimator. In this way, the procedure of computing EUP via Monte Carlo approximation can be seen to be equivalent to a statistical version of Sensitivity Analysis.



(a) Scenario evaluation



(b) Sample evaluation with optional measurement

Figure 5: Comparison of traditional Sensitivity Analysis scenario evaluation with sample evaluation procedure of generalised VoIA

The generality of the procedure for computing the utility performance of a given control scheme for a given Decision System is important, as it enables the generalised VoIA framework to be applied to simulation based problems, greatly broadening its potential Engineering applications. These simulations could take the form of Digital Twins of infrastructure assets and systems, allowing the framework to be used to assess the 'performance robustness' of the corresponding Physical Twins, and hence provide a quantification of some aspect of their resilience.

4. Linear Programming Based Decision Problems

4.1 Motivation

As outlined in Section 2, the solution of capacity expansion planning problems to identify minimal cost fully renewable energy system design strategies is of critical importance to enabling a cost-effective transition to net zero carbon future energy systems. Current state-of-the-art methods, such as [20–22], produce linearised models of the temporal network physics of energy systems, and from these formulate Linear Programs (LPs) that are solved to identify the minimal cost system design under the model. As discussed in Section 2.1.i), energy systems are subject to substantial uncertainties, which leads to uncertainty in the parameters used to define these Energy System Optimisation Models (ESOMs). However, existing LP-based methods for large-scale energy system analysis are all deterministic, using point estimates of the uncertain parameters. They can therefore be seen to be Expected Value Problems (EVPs) for the capacity expansion planning strategy selection Bayesian Decision Problem (BDP).

This motivates the use of the generalised VoIA framework proposed in Section 3.4.ii) to study the impact that the underlying uncertainties of energy systems have on the performance of the system designs suggested by these LP-based ESOMs.

The remainder of the Section presents how the generalised VoIA framework can be applied to general LP-based decision problems.

4.2 One-Stage Problem

It is desired to formulate a one-stage Bayesian Decision Problem (BDP) for a linear system in which the following general Linear Program can be naively defined,

$$\begin{aligned} \max_x \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \end{aligned} \tag{11}$$

where x is the action, $c^T x$ the objective, and $Ax \leq b$ the constraint set of the system. However, the parameters defining the system are all uncertain, $c, A, b \sim \pi(c, A, b)$.

4.2.i) Uncertain Constraint Set & Infeasibility

Once the action, x, has been taken, the true state of the world, $\theta = \{c, A, b\}$, is revealed. However, due to the uncertainty in the constraint set, the taken action x may be infeasible.

For systems where infeasibility of the action is not tolerable, then solution strategies must use the 'safe' system constraint set, which is the subset of the uncertain constraint set in which all points are feasible for all possible realisations of A, b, defined $\mathcal{S} = \{x : Ax \leq b \forall\ A, b \in \Omega\}$ where Ω is the sample space of the stochastic problem. This approach is applied in [67, 84]. However, this is a severe modelling approach which can lead to excessively conservative and poorly performing solutions. Further, for this case, unbounded probability distribution models such as the Gaussian lead to the BDP, Equation (11), being ill-defined, i.e. without a solution.

In some Engineering applications infeasible solutions may be allowable, but it may be desired to limit the probability of their occurrence. In this instance, Chance Constrained Programming (CCP) [74, 106, 107] can be employed, leading to the optimisation formulation,

$$\begin{aligned} \max_{x} \quad & c^T x \\ \text{s.t.} \quad & \Pr\left(Ax \leq b\right) \geq 1 - \gamma \end{aligned} \tag{12}$$

where γ is the constraint violation probability tolerance parameter. However, unless certain conditions are satisfied by the problem formulation, the Chance Constrained optimisation becomes either non-linear, or more problematically non-convex [108, 109]. Therefore, CCP does not allow for the study of general uncertainties in linear systems, and so an alternative approach is sought.

4.2.ii) Constraint Violation Penalisation

In the proposed approach it is assumed that action infeasibility is allowable, but that there is some cost associated with violating the constraints of the system. Therefore, the system utility function is extend to be defined as,

$$u(x, \theta) = c^T x - f\left(Ax - b\right) \tag{13}$$

where $f(\cdot)$ is the constraint violation penalisation cost function. The overall utility can be seen to have contributions from the objective of the linear system and from the constraint violation penalty, which can be more explicitly expressed as $f(\max[Ax - b, 0])$, where the $\max$ is a vector operation. Therefore, this formulation can be seen to be a relaxation of the original Linear Program. For some applications there may be a subset of deterministic constraints, $Cx \leq d$, which cannot be violated, and these can be additionally imposed on the system.

The BDP for the system can therefore be written as,

$$\begin{aligned} \max_{x} \quad & \mathbb{E}_{c,A,b}\left\{c^T x - f\left(\max[Ax-b,0]\right)\right\} \\ \text{s.t.} \quad & Cx \leq d \end{aligned} \tag{14}$$

which is not in general a LP, due to both the constraint violation penalisation function, $f(\cdot)$, and the probability density functions of the uncertain parameters. Hence, the BDP is a non-linear Stochastic Program.

For computational simplicity the expectations over the uncertain parameters can be separated out, leading to,

$$\begin{aligned} \max_{x} \quad & \mathbb{E}_{c}\{c\}^T x - \mathbb{E}_{A,b}\left\{f\left(\max[Ax-b,0]\right)\right\} \\ \text{s.t.} \quad & Cx \leq d \end{aligned} \tag{15}$$

If a linear constraint violation penalty is chosen, defined using a per unit violation distance cost vector v, the utility function of the system becomes,

$$u(x,\theta) = c^T x - v^T \max[Ax-b,0] \tag{16}$$

This can be seen to be closely related to the stochastic Lagrangian function of the naively proposed LP of the system[10], showing this to be a very natural way of penalising constraint violations in the context of a linear system.

Alternatively, an indicator function based constraint violation cost could be imposed,

$$u(x,\theta) = c^T x - v^T \, \mathbb{1}(Ax-b>0) \tag{17}$$

[10] The Lagrangian of Equation (11) is $\mathcal{L}(x,\mu) = c^T x - \mu^T(Ax-b)$, where $\mu \geq 0$ and the complementary slackness condition for optimality, $\mu^T(Ax-b)=0$, holds. Specifically, the utility function is an evaluation of the stochastic Lagrangian function at the point in dual space defined by the cost vector v, i.e. $\mathcal{L}(x,\mu=v)$.

This can be seen to impose a cost on the marginal probability of violation of each constraint, as,

$$\mathbb{E}_{A,b}\left\{v^T\,\mathbb{1}(Ax-b>0)\right\} = \sum_i v_i \mathbb{E}_{A,b}\left\{\mathbb{1}\left((Ax-b)_i>0\right)\right\} = \sum_i v_i \Pr\left((Ax-b)_i>0\right) \quad (18)$$

and is therefore closely linked with a relaxation of Chance Constrained programming.

The constraint violation penalisation function for the BDP, $f(\cdot)$, should be chosen to most appropriately model the physical cost incurred by the violation of the system constraints, and its functional form can be selected for each constraint individually as to be most suitable. The form of cost function has a negligible impact on the computational cost of the EVPI calculation (Section 4.2.iv)), but significantly influences the complexity of determining the stochastically optimal solution to the BDP, required to compute the VSS (Section 4.2.v)).

4.2.iii) Expected Value Problem

The EVP for the uncertain linear system is given by the LP,

$$\begin{aligned} \max_x \quad & \hat{c}^T x \\ \text{s.t.} \quad & \hat{A}x \leq \hat{b} \end{aligned} \quad (19)$$

where $\hat{c} = \mathbb{E}\{c\}, \hat{A} = \mathbb{E}\{A\}, \hat{b} = \mathbb{E}\{b\}$, noting that the EVP solution cannot violate its estimation of the constraint set, simplifying the utility function. The solution to this LP, $\hat{x}$, is used as an approximation to the stochastically optimal (Prior) solution, x^*.

The expected utility performance achieved by the EVP solution can be computed via Monte Carlo approximation, using samples drawn from the parameter distributions, $\{(c^{(j)}, A^{(j)}, b^{(j)}) \sim \pi(c, A, b)\}_{j=1}^N$,

$$\hat{y} = \mathbb{E}_{c,A,b}\left\{u(\hat{x},\theta)\right\} \approx \frac{1}{N}\left(\left(\sum_j c^{(j)}\right)^T \hat{x} - \sum_j f\left(\max\left[A^{(j)}\hat{x} - b^{(j)}, 0\right]\right)\right) \quad (20)$$

4.2.iv) Expected Value of Perfect Information

As described in Section 3.4.ii), the EVPI can be used as a metric to quantify the impact the uncertainty of the model parameters has on the performance of the LP-based decision process.

For the case of LP-based decision models, the expected utility of the Perfect Information Pre-Posterior decision process is computed by solving the LP defining the system for each sample from the parameter distributions,

$$\begin{aligned} y_p^{**(j)} = \max_x \quad & {c^{(j)}}^T x \\ \text{s.t.} \quad & A^{(j)} x \leq b^{(j)} \end{aligned} \tag{21}$$

and using the mean utility as a Monte Carlo approximation of the true expected utility,

$$y_p^{**} = \mathbb{E}_{c,A,b} \left\{ u(x_p^{**}, \theta) \right\} \approx \frac{1}{N} \sum_j y_p^{**(j)} \tag{22}$$

In this case the utility simplifies to the returned objective function value, as the LP cannot violate the system constraints as it has knowledge of the true constraint set. Note that insufficient penalisation of constraint violations will result in ill-conditioned BDPs for which the optimal Perfect Information action is infeasible and cannot be identified by the above LP.

The Expected Value of Providing Perfect Information to the linear Program decision process (EVPPIP) is therefore given by,

$$\text{EVPPIP} = y_p^{**} - \hat{y} = \text{EVPI} + \text{VSS} \tag{23}$$

and is interpreted as the reduction in the expected utility obtained by the LP-based decision process as a result of the uncertainty in the system parameters, i.e. how much worse the LP performs in expectation due to not having knowledge of the true system parameter values, and instead having to use their expected values. In this way the EVPPIP acts a UQ metric quantifying the 'performance robustness' of the LP-based decision process to the underlying uncertainty.

The EVPIIP $=$ EVII $+$ VSS can also be computed and used to aid assessment of the net benefit of improved measurement in the manner of classical VoIA.

4.2.v) Value of Stochastic Solution

If the BDP of the system, given by Equation (14), can be solved to determine the stochastically optimal Prior action, or this solution can be closely approximated, then the VSS can be computed as,

$$\text{VSS} = y^* - \hat{y} \tag{24}$$

where y^* is the expected utility obtained by the (approximate) stochastically optimal solution.

The VSS provides a more realistic metric of the decision performance reduction arising from the underlying uncertainty of the system, as the decision maker is not able to achieve y_p^{**} unless they have access to perfect measurements of the true system parameters (in which case the uncertainty analysis is redundant), but they are in theory capable of implementing the stochastically optimal Prior action and thus achieving y^*.

However, solving the Prior SDP to identify the stochastically optimal Prior action, and hence y^*, may be computationally intractable due to the non-convexity and potential ill-conditioning of the BDP Stochastic Optimisation, Equation (14). Stochastic global optimisation algorithms such as Stochastic Gradient Descent [110], Particle Swarm, or Evolutionary Strategies [111, 112], could be used to approximately solve the Stochastic Optimisation. Alternatively, duality bound methods could be employed to compute an upper bound on the VSS [113]. Monte Carlo approximation of the EVPPIP $=$ EVPI $+$ VSS requires the solution of the Perfect Information LP, Equation (21), for each of the N samples drawn from $\pi(\theta)$, and therefore can be used as a comparatively computationally efficient upper bound on the VSS, as its evaluation requires only the solution of deterministic decision problems.

If the VSS, or its upper bound, can be demonstrated to be small, then the EVP can be seen to be 'performance robust' to the underlying uncertainty of the problem, and thus provide a sufficiently performative and highly computationally efficient approximation of the stochastically optimal solution. In this way, the generalised VoIA framework can be used to assess the performance suitability and robustness of deterministic LP-based decision processes.

4.2.vi) Illustration of Methodology

Figure 6 provides a two-dimensional illustration of the machinery of the proposed methodology for the purposes of providing intuition on the problem setup and motivation behind the formulation. The assumed constraint set of the EVP is shown using black lines, with the EVP solution indicated with a black spot. Grey lines show the true constraint set for two samples from the distribution of the uncertain system parameter, θ, with one sample leading to the EVP solution violating the true system constraints. The Perfect Information Posterior solutions for the two samples are marked. The stochastically optimal Prior solution is indicated in red, and is a conservative due to the penalisation of constraint violations.

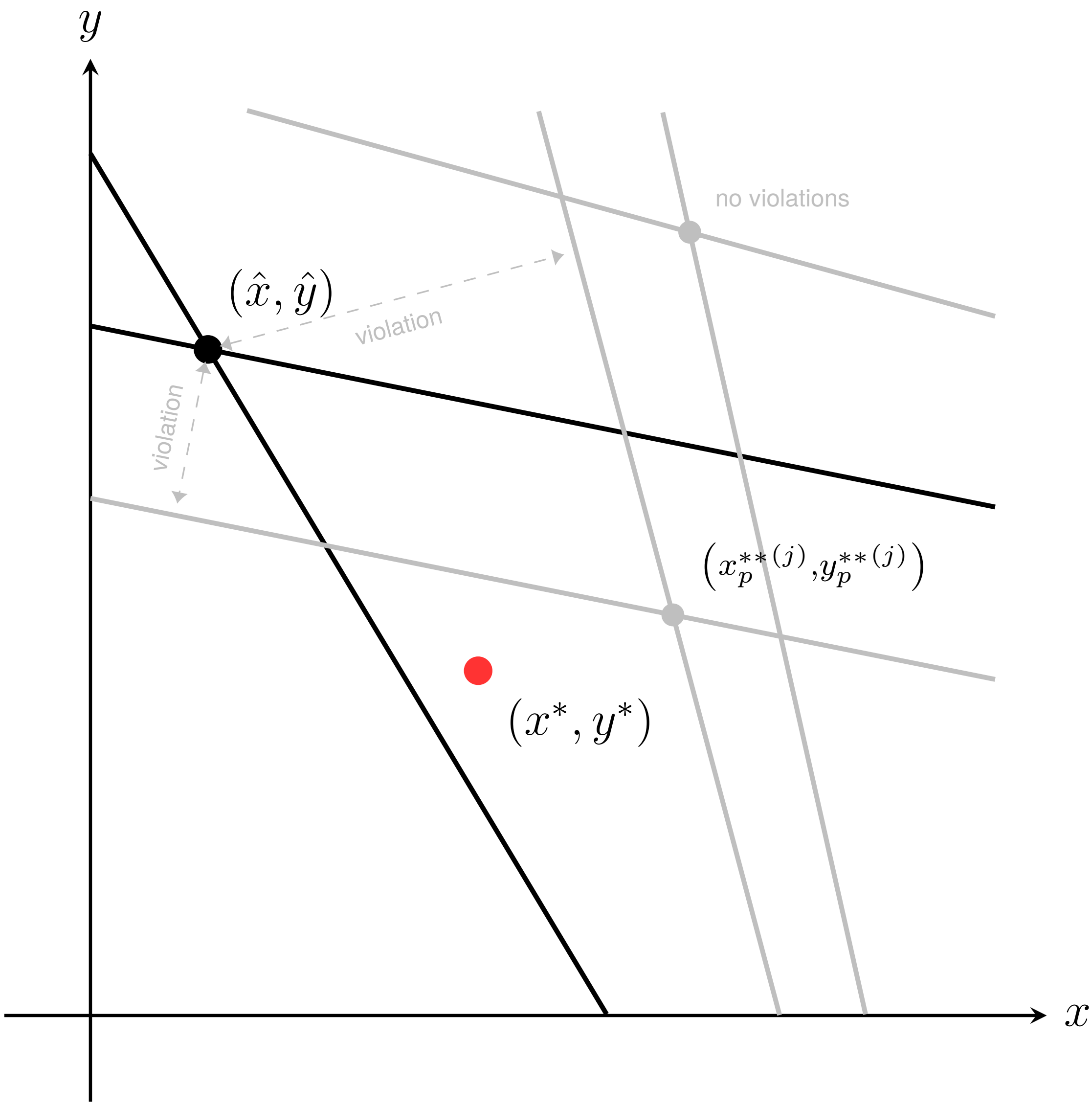


Figure 6: Illustration of constraint violations of EVP solution for system parameter samples

4.2.vii) Extension to Action & Constraint Violation Cost Uncertainty

The proposed method can be readily extended to study the impact of action implementation uncertainty and constraint violation cost uncertainty on the performance of LP-based decision processes, as demonstrated in Appendix E.1.

4.3 Two-Stage Problem

The proposed methodology for studying the impact of uncertainties on LP-based decision problems can be readily extended to the analysis of two-stage decision problems. Such two-stage problems are better suited for application to the study of large-scale ESOMs, such as [20–22], which have a two-layer optimisation structure, where decisions are made on both the high-level design of the energy system and the operational strategy of the system. The analysis of two-stage problems behaves very similarly to the one-stage analysis and provides limited additional insight, hence details of the extension are provided in Appendix E.2 for reference only.

5. Numerical Example of Application of VoIA to LP-based Energy System Model

This Section presents a simple numerical example of the application of the uncertainty analysis methodology for LP-based decision problems proposed in Section 4 to a stylised energy system capacity expansion planning problem. Through this example, the capabilities of the proposed analysis, and its potential for providing insight into the impact of uncertainties in energy system design problem applications, are demonstrated.

5.1 Definition of Stylised Energy System Design Task

A stylised renewable generation asset portfolio design task is considered, where it is desired to identify the minimal cost set of renewable generation asset capacities to meet the energy needs of the North East of England, subject to operational conditions restricting the allowable variability of the net power generation timeseries, considering only supply side effects and neglecting transmission losses. This results in a linear model of the NE energy supply system.

The naive stochastic Linear Program for the system is given by,

$$\min \quad \sum_i c_i C_i \tag{25}$$

$$\text{over} \quad C_i \qquad \forall\, i$$

$$\text{s.t.} \quad \sum_{i,t} C_i \tilde{g}_i(t) \geq \alpha \tag{25a}$$

$$\sum_t \max\Big[0, \beta - \sum_i C_i \tilde{g}_i(t)\Big] \leq \gamma \tag{25b}$$

$$C_i \geq 0 \qquad \forall\, i \tag{25c}$$

where the parameters of the problem are given as follows:

C_i : asset capacities of the three generation technologies, solar, offshore wind, and nearshore wind, located as shown in Figure 7

$\tilde{g}_i(t)$: historic hourly resolved normalised generation power timeseries for each asset, obtained from renewables.ninja [38, 39], over the time window considered, $t \in \mathcal{T}$

c_i : uncertain annual per unit capacity costs of each generation technology

α : minimum annual TWhs of energy generation required from the generation portfolio, parameterising the aggregate energy generation constraint Equation (25a), which can also be specified as an equivalent mean power generation in GW

β, γ : parameters of the stylistic net generation variability constraint, Equation (25b), which requires fewer than γ GWh/year to be dropped below a threshold power generation level of β GW

The non-negativity of asset capacity constraints, Equation (25c), are deterministic constraints for the problem and cannot be violated.

Figure 7: Location of renewable generation assets for NE energy system design task [114]

A linear constraint violation penalisation cost function is used, resulting in the following formulation of the BDP for the energy system design task,

$$\begin{aligned} \min \quad & \mathbb{E}_{c,\theta}\Bigg\{ \sum_i c_i C_i + \Bigg(v_t \cdot \max\Big[0, \alpha - \sum_i C_i \sum_t \tilde{g}_i(t)\Big] \\ & \qquad + v_d \cdot \max\Big[0, \sum_t \max\Big[0, \beta - \sum_i C_i \tilde{g}_i(t)\Big] - \gamma\Big]\Bigg)\Bigg\} \end{aligned} \tag{26}$$

$$\text{over} \quad C_i \qquad \forall\, i$$

$$\text{s.t.} \quad C_i \geq 0 \qquad \forall\, i \tag{26a}$$

where θ is a random variable parameterising the uncertainties in $\tilde{g}_i(t)$, as described later, and noting that as the objective is to be minimised the constraint violation cost is additive.

The EVP of the BDP is therefore given by,

$$\min \quad \sum_i \hat{c}_i C_i \tag{27}$$

$$\text{over} \quad C_i \qquad \forall\, i$$

$$\text{s.t.} \quad \sum_{i,t} C_i \widehat{\tilde{g}_i(t)} \geq \alpha \tag{27a}$$

$$\sum_t \max\left[0, \beta - \sum_i C_i \widehat{\tilde{g}_i(t)}\right] \leq \gamma \tag{27b}$$

$$C_i \geq 0 \qquad \forall\, i \tag{27c}$$

This can be converted into explicit LP form by applying the standard trick for handling convex $\max[\ldots]$ formulations [115], introducing slack variables ϕ_t to represent $\max\left[0, \beta - \sum_i C_i \tilde{g}_i(t)\right]$, yielding,

$$\min \quad \sum_i \hat{c}_i C_i \tag{28}$$

$$\text{over} \quad C_i, \phi_t \qquad \forall\, i, t$$

$$\text{s.t.} \quad \sum_{i,t} C_i \widehat{\tilde{g}_i(t)} \geq \alpha \tag{28a}$$

$$\sum_t \phi_t \leq \gamma \tag{28b}$$

$$\phi_t \geq \max\left[0, \beta - \sum_i C_i \widehat{\tilde{g}_i(t)}\right] \qquad \forall\, t \tag{28c}$$

$$\phi_t \geq 0 \qquad \forall\, t \tag{28d}$$

$$C_i \geq 0 \qquad \forall\, i \tag{28e}$$

5.2 Numerical Trials & Results

The EVP was solved using the parameter values given in Table 3, and hourly normalised generation data from 2018. A summary of the identified solution, $\hat{x}$, is provided in Table 4.

Mean annualised asset capacity costs[11] (£M/GW/year)			System constraint parameters		
Solar	Offshore wind	Nearshore wind	α (GW)	β (GW)	γ (GWh/yr)
150	400	350	10	2	438[12]

Table 3: Parameter values used for Expected Value Problem

Asset capacities (GW)			Estimated system cost (£B/yr)
Solar	Offshore wind	Nearshore wind	
0.79	13.57	3.82	6.884

Table 4: Summary of solution to Expected Value Problem

Uncertainty analysis was performed considering uncertainties in the asset costs and normalised generation power timeseries, with the renewable generation uncertainty initially modelled via the inclusion of a multiplicative random variable representing uncertainty in the capacity factor of generation for each asset[13],

$$g_i(t) = \kappa_i \widehat{\tilde{g}_i(t)} \qquad \text{where } \kappa_i \sim \pi(\kappa_i) \tag{29}$$

[11] Annualised asset capacity costs approximated via back calculation using costs and service lifetimes from [116]

[12] Equivalent to an output of 1GW below the threshold β for 5% of the year

[13] Note, this very simple uncertainty model leads to non-physical normalised generation powers, i.e. normalised values above 1, however for the purposes of this stylistic problem this is acceptable

For simplicity, the random variables defining the underlying uncertainties of the energy system were taken to be Gaussian distributed as,

$$c_i \sim \mathcal{N}\left(\hat{c}_i, 10^2\right) \qquad \kappa_i \sim \mathcal{N}\left(1, 0.01^2\right) \tag{30}$$

Taking the constraint violation penalisation costs to be,

$$v_t = 200 \text{ £M/GWh} \qquad v_d = 500 \text{ £M/GWh} \tag{31}$$

the true expected utility (cost) of the EVP solution, $\hat{x}$, was approximated via Monte Carlo approximation using 1000 samples from the uncertain parameter distributions.

Then, the expected costs of the Perfect Information & Partial Perfect Information Pre-Posterior decision processes were computed by solving the posterior decision LP for the cases where perfect information on the: asset costs, asset capacity factors, and both asset costs and capacity factors; were available. For the Partial Perfect Information decision problem, the unknown parameter values were taken as their mean values to preserve the determinism of the LP. For each expected cost approximation 100 samples[14] from the parameter distributions were used.

Finally, an attempt was made to identify the stochastically optimal Prior system design strategy by directly optimising the expected cost function using Particle Swarm Optimisation (PSO).

The results of these numerical trials are summarised in Table 5.

A further uncertainty analysis was conducted, analysing the impact of uncertainty in (only) the renewable power generation timeseries on the performance of the EVP LP-based system design strategy. Each year of available data from 2000-2019 was considered as a draw from the underlying distribution of power generation timeseries, with the EUP of the EVP and Perfect Information Pre-Posterior decision processes evaluated for each of the year samples to compute the EVPPIP. Figure 8 and Table 6 present the results of this uncertainty analysis, in which the EVPPIP was found to be 0.243 £B/yr, or 3.46% of EVP solution EUP, $\hat{y}$.

[14] A small number of samples were used as the computational cost of solving the necessary LPs was significant and limited computational resources were available. It is recommended that more samples are used in full-scale studies to reduce the Monte Carlo approximation error. However, the computational expense of this uncertainty analysis method remains a significant consideration, as discussed in Section B

[15] Achieved by generation asset portfolio: 1.38 GW Solar, 13.19 GW Offshore wind, 7.80 GW Nearshore wind

Metric	Value (£B/yr) [% of †]
EVP estimated cost (objective)	6.884
EVP true expected cost †	**6.942**
Asset cost informed decision expected cost	6.914
Asset capacity factor informed decision expected cost	6.881
Asset cost & capacity factor informed decision expected cost	**6.864**
Expected cost of best solution identified by PSO	**7.038**[15]
Expected value of Partial Perfect Information, costs	0.0283 [0.411%]
Expected value of Partial Perfect Information, capacity factors	0.0608 [0.883%]
Expected value of Perfect Information, costs & capacity factors	0.0774 [1.124%]

Table 5: Summary of cost & capacity uncertainty analysis results

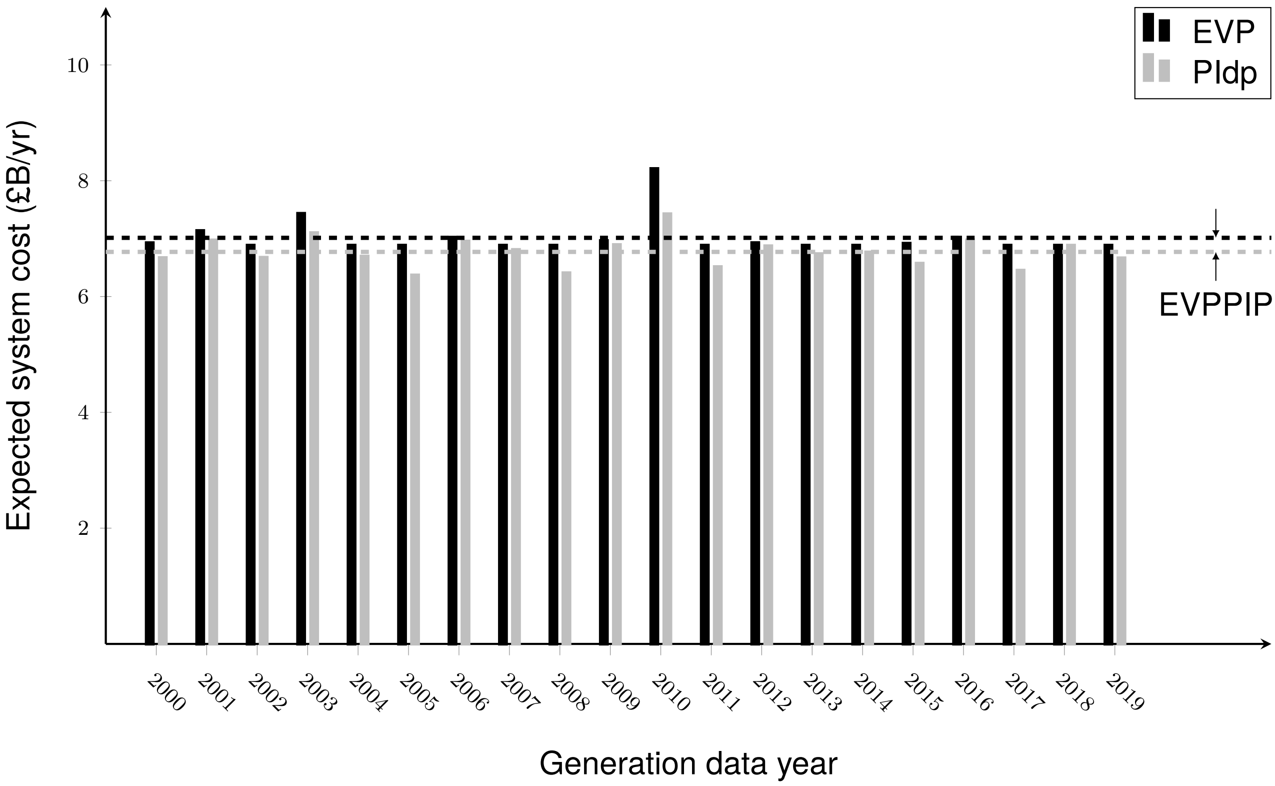


Figure 8: Summary of renewable generation timeseries uncertainty analysis results

Generation data year	Expected cost of EVP (£B/yr)	Expected cost of Perfect Information decision process, PIdp (£B/yr)
2000	6.927	6.670
2001	7.137	6.975
2002	6.884	6.675
2003	7.434	7.101
2004	6.884	6.695
2005	6.884	6.370
2006	7.022	6.955
2007	6.884	6.810
2008	6.884	6.408
2009	6.964	6.895
2010	8.208	7.427
2011	6.884	6.514
2012	6.928	6.874
2013	6.884	6.738
2014	6.884	6.767
2015	6.918	6.573
2016	7.023	6.976
2017	6.884	6.452
2018	6.884	6.884
2019	6.884	6.666
Mean	7.014	6.771

Repeated values correspond to instances in which the EVP solution is feasible and so does not incur any constraint violation penalty.

Table 6: Summary of renewable generation timeseries uncertainty analysis results

5.3 Implications for Energy Systems Modelling

This numerical example has demonstrated the capabilities of the uncertainty analysis methodology for LP-based decision problems proposed in Section 4 in its application to the study of energy systems problems, and the quantification of the impact of underlying uncertainties. The cost of the considered generation asset cost and capacity factor uncertainties was shown to be of the order 1% of total system cost, and that of renewable generation timeseries uncertainty, to be approximately 3.5%.

These results motivate more rigorous study of the impacts of uncertainties on the performance of state-of-the-art LP based ESOMs, and the need for uncertainty informed models. Future research effort should be applied to improve upon these initial findings. Such studies should use data-driven methods for selecting distributions of the underlying system uncertainties, and perform sensitivity analysis to explore the effect of modelling choices on the results of the VoIA. Further, this form of analysis could be used to study the modelling benefits of including additional data in ESOMs through the use of longer historic time windows, allowing for the identification of the optimal trade-off between computational cost and solution performance.

6. Conclusion & Future Works

This work has shown the potential of VoIA as a method for providing insight into the significance of underlying uncertainties in energy systems problems, and the importance of developing uncertainty informed decision models, through its ability to quantify the impact of uncertainty on the solutions to decision problems in terms of the decision objective. It was thereby demonstrated that VoIA can be used to support meta decision making tasks within energy systems, such as the determination of optimal measurement strategies, the targeted reduction of uncertainties through research or improved sensing, and the identification of optimal model data usage.

Further, extensions to the classical VoIA framework were presented which greatly increase the diversity of problems to which the method can be applied to analyse the impacts of uncertainty. Additionally, these extensions allow VoI metrics to be exploited as Uncertainty Quantification metrics that inform meta decision makers of the ‘performance robustness’ of different decision processes for a given system, enabling the impacts of uncertainty to be considered alongside expected performance in their comparison.

Existing applications of VoIA in the energy systems literature are limited in both their scope and range of problems considered, and the capabilities of the VoIA framework highlighted through this work warrant significant further research effort in this area. Further exploration of the types of energy system problems to which VoIA can be applied is required, to identify those for which the framework can make contributions to the understanding of the behaviour of energy systems under the uncertainty imposed by the integration of variable renewable generation. Some potential applications for future study are proposed in Appendix A.

The extensions to the classical VoIA framework proposed in this piece provide a starting point for the use of the VoIA methodology to study more complex energy systems decision problems, and provide improved measures of the impact of uncertainties on practical decision processes. Future works are required to exploit the potential of these generalisations to provide insight into how uncertainties affect energy system problem applications, and further development of the generalised methodology is required to identify additional capabilities of the framework. Of particular importance is the investigation of the use of the generalised framework for studying the impacts of uncertainty on Reinforcement Learning based controllers, as a rigorous methodology for such analysis would allow for the study of far more general systems, without the requirement for closed form expressions for both the system behaviour and control scheme. This analysis capability would open up opportunities to perform uncertainty analyses on blackbox simulation based systems for which only state-action-reward information is available, which arise in a wide range of Engineering applications. This form of VoIA generalisation may also enable the assessment of the impact of system and measurement uncertainties on the performance of automated decision processes within Digital Twin ecosystems. Due to the ability of the VoIA methodology to treat uncertainty in an end-to-end manner, and quantify its impact on the overall performance of a system, the framework can be used to analyse the effect of uncertainties on both the isolated performance of an individual component within a Digital Twin, and on the performance of complex, interdependent networks of Digital Twins. Internal interdependencies and resulting uncertainty propagation are incorporated into the derived uncertainty metric, which can be used to assess the critical uncertainties with respect to the performance of decision problems within the system, and the suitability of deterministic Digital Twin models and the corresponding need for the development of 'uncertainty informed' Digital Twins.

The example application of the LP-based decision problem uncertainty analysis methodology proposed in Section 4, presented in Section 5, demonstrates the types of uncertainty analysis that can be performed on state-of-the-art Energy System Optimisation Models, such as [20–22], using the generalised VoIA methodology. The preliminary results motivate further research into the critical uncertainties affecting such ESOMs, and the suitability of deterministic linear models for identifying low cost future decarbonised energy system designs that are robust to the underlying certainties affecting their development and operation. Understanding the comparative expected performance of these designs, and those recommended by far more computationally expensive stochastic energy system optimisation methods, is critical. This understanding would enable researchers to determine whether existing deterministic linear models can provide policy makers with reliable energy system design recommendations, or, whether further research effort is required to develop computationally tractable stochastic methods for large-scale energy system models. Application of the generalised VoIA methodology to high-resolution, full-scale ESOMs, and rigorous analysis of the impacts of the uncertainties associated with energy systems on these models, will be required before definitive conclusions can be drawn in this regard.

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Appendices

A. Further VoI Applications to Energy System Problems

In this Appendix a series of potential applications of VoIA to energy systems problems are briefly presented to demonstrate the diversity of problems that can be studied using the methodology, and the breadth of different insights about uncertainty and measurements that can be gained from its application.

A.1 VoI for Measurement to Improve Predictions in Power Dispatch Scheduling

In power dispatch scheduling problems, both the available power generation from renewable sources and the demanded power in future time instances are uncertain. This uncertainty affects the performance of look-ahead scheduling algorithms which must base their decisions on uncertain predictive models of the supply and demand powers, as realisations of power volumes that do not match the implemented schedule lead to mismatches of supply & demand that are costly to correct through grid-balancing mechanisms (purchasing power on the spot market is far more costly than correctly predicting requirement and purchasing it on the day-ahead market).

Measurement of power volumes at a given instance reduce the uncertainty of predicted values in future time instances, for example through the updating of a Bayesian predictive distribution. However, deploying real-time telemetry systems across a power network incurs a significant cost.

VoIA could be applied to compute the expected value of telemetric measurements at different sampling frequencies for the reduction of uncertainty in predictive distributions for generation and power powers, for a given scheduling algorithm applied to a power dispatch optimisation problem. This would allow for the optimal usage of telemetry data for scheduling to be identified.

Note that the VoI will depend on the state of the power system, i.e. the power levels, available reserve, etc.. This uncertain state must either be included in the expectation calculation, or the VoI field over the system state could be computed to determine the optimal measurement frequency for each sub-region of the state space so that a dynamic optimal sampling frequency can be employed.

A.2 Determining Optimal Data Usage in Energy Systems Models

As briefly mentioned in Section 5.3, VoIA could be applied to determine the trade-off between solution 'performance robustness' and computational expense for ESMs so that the optimal data usage can be identified.

Applying more data to ESMs by analysing longer historic operational condition time windows provides the models with more information on the underlying distribution of operational conditions, and therefore (likely but not provably) allows them to identify system design solutions with improved expected performance. However, the computational complexity of standard Interior-Point Method LP solvers is $O(n^3)$ [117], meaning that computational expense grows rapidly with the length of time window analysed.

VoIA could be applied to such problems to determine the trade-off between expected solution performance and computational expense of model evaluation, to allow researchers to identify the optimal data usage for ESM application studies.

A.3 VoI for Consumer DSR Price-Volume Curves for Grid Operators

Demand-Side Response (DSR) will be a key mechanism for supply-demand power matching in future high-penetration renewable power systems. Whilst for industrial and retail loads DSR contracts can be agreed between the load and grid operator, this is unlikely to be feasible for consumers, who will likely desire more flexibility in their energy usage. Therefore, if consumer DSR is to be used, there will be significant uncertainty in the consumer demand response to a given price signal at a given time. This results in uncertainty in the resulting net demand load for grid operators attempting to use consumer DSR for supply-demand matching in their dispatch scheduling strategies. As in Section A.1 this uncertainty reduces the expected performance of scheduling algorithms.

Trials and surveys can be carried out to reduce the uncertainty of consumer DSR price response curves, the uncertain model mapping DSR market price to demand adjustment, which will exhibit significant system state dependence. Therefore, VoIA can be used to determine whether such costly trials should be carried out to improve the predictive models of consumer DSR, or whether prior models obtained from historic data are sufficient for the purposes of dispatch scheduling.

A.4 VoI for EV Peak Shifting Capacity for Grid Operators

In future power systems it is likely that distribution grid connected EVs will be leveraged as effective dispatchable power for supply-demand matching, in a similar fashion to consumer DSR as discussed in Section A.3. However, at any given time the number of EVs available to provide peak-shifting services in a given distribution region, determining the available power capacity, and the state-of-charge of those EVs, determining the available energy capacity, could be unknown to the grid operator and thus uncertain.

Telemetry systems could be installed to provide grid operators with this EV-based power dispatch & energy arbitrage capacity information, either for all EV charging points in the distribution region (giving perfect information), or for some sub-sample of charging points (providing imperfect information via a Monte Carlo estimate). Therefore, VoIA studies could be performed to determine whether such telemetry systems are economical for the grid operator, and if so, the optimal proportion of metered charging points.

Note, similar studies could be performed for domestic battery storage assets if they become significant actors in power distribution systems.

A.5 VoI for Battery Operating Characteristics for Power Support Service Provides

Battery systems are commonly used to provide primary and secondary power balancing services to improve grid stability. Battery operators looking to provide these services seek to optimise the scheduling of power dispatch from their storage units to maximise the profitability of the assets. This profitability is strongly influenced by the operating characteristics of the batteries at the time of power dispatch, such as efficiency, state of charge, and self-discharge rate, and many of these operational characteristics are impacted by environmental conditions, particularly ambient temperature. Therefore, due to uncertainties in weather predictions and battery monitoring system measurements, the operating characteristics of the battery assets for a given scheduling duration will be uncertain, and as a result there will be uncertainty in performance of a given scheduling strategy.

VoIA can be used to determine whether deployment and/or improvement of monitoring systems for battery operating conditions and ambient weather conditions provide a net profitability improvement for power support service providers.

B. Computational Considerations of VoIA

Whilst VoIA provides important information on the impact of uncertainties on the performance of decision processes in the context of a given decision problem, this information comes at a significant computational cost, which limits the scope of tractable decision problems that can be analysed using the methodology.

For the purposes of this discussion on the computational considerations of VoI calculations, the stages of the decision process evaluation will be separated so that the solution of the decision problem is considered as the evaluation of an 'action model' (AM) to determine the action taken by the decision process, followed by the evaluation of a 'utility model' (UM) to determine the utility achieved by that action given a certain realisation of the uncertain parameters of the system. This distinction is made to allow for a more clear discussion of the generalised VoIA framework, however it should be noted that for classical, decision tree based VoIA, evaluation of the AM involves evaluation of the expectation over θ of the UM for each action in the action space, $a \in \mathcal{A}$, and the selection of the expected utility maximising action from that set, hence the computational cost of AM is $|\mathcal{A}|$ times that of the expected UM calculation, and the UM does not then need to be re-evaluated.

For many practical decision problems, such as the numerical energy systems example discussed in Section 5, the AM has a significantly higher computational cost than the UM. However, one of the key advantages of the generalised VoIA framework is that it allows for the application of simple control schemes to highly complex systems, such as simulation based models, for which evaluation of the AM has a far lower computational expense than for the UM.

Considering initially the evaluation of the expected utility of a given action over the uncertain parameter θ via Monte Carlo approximation,

$$y_\theta(a) = \mathbb{E}_\theta\left\{u(a,\theta)\right\} \approx \frac{1}{N}\sum_{j=1}^{N} u\big(a,\theta^{(j)}\big) \quad \text{where } \theta^{(j)} \sim \pi(\theta) \quad \forall\, j \tag{32}$$

This can be seen to have a computational cost of N times that of the UM, where N is the number of samples from $\pi(\theta)$ required to achieve an acceptable Monte Carlo approximation error.

Therefore, determining the EUP of the EVP has a computational cost of,

$C(\text{AM}) + N \cdot C(\text{UM})$, where the function $C(\cdot)$ indicates the computational cost of a model. Already this simple calculation becomes highly computationally expensive for problems with large $C(\text{UM})$, such as large-scale simulation based models. Hence for such problems, the use of VoIA may be constrained to the comparison of EUP for different controller architectures. Though, it should be noted that the N UM evaluations are readily parallelisable.

For the case of a control scheme which has access to measurement information, $a(z, \ldots)$, its EUP is computed as,

$$y_{\theta,z}\left(a(z,\ldots)\right) = \mathbb{E}_{\theta,z}\left\{u\left(a(z,\ldots),\theta\right)\right\} = \mathbb{E}_{z}\left\{\mathbb{E}_{\theta|z}\left\{u\left(a(z,\ldots),\theta\right)\right\}\right\} \tag{33}$$

The second expression requires the nested evaluation of Monte Carlo approximations, resulting in a computational expense of $N \cdot C(\text{AM}) + N^2 \cdot C(\text{UM})$, as for each sample from $\pi(z)$ the associated control action must be evaluated as well as a Monte Carlo approximation of the corresponding expected utility.

However, if the uncertainty model of the system is such that samples from the joint distribution $\pi(\theta, z)$ can be obtained, where the conditional distributions of the joint are the required posteriors $\pi(\theta|z)$, then a single combined Monte Carlo estimate can be used to approximate the first expectation expression directly. This reduces the computational cost of evaluating $y\left(a(z,\ldots)\right)$ to $N \cdot \left(C(\text{AM}) + C(\text{UM})\right)$.

For the case of uncertain measurements,

$$y_{\theta,z,z'}\left(a(z,\ldots)\right) = \mathbb{E}_{\theta,z,z'}\left\{u\left(a(z,\ldots),\theta\right)\right\} = \mathbb{E}_{z'}\left\{\mathbb{E}_{z|z'}\left\{\mathbb{E}_{\theta|z}\left\{u\left(a(z,\ldots),\theta\right)\right\}\right\}\right\} \tag{34}$$

which involves three nested layers of Monte Carlo approximations, inducing a computational cost of $N^2 \cdot C(\text{AM}) + N^3 \cdot C(\text{UM})$. Though again, if the necessary joint distribution can be sampled from, the computational cost can be reduced significantly.

Hence, the computational cost of VoIA analysis can be seen to be vastly greater than that of a point evaluation via a deterministic form of the decision problem, such as a single evaluation of the EVP AM and UM, as well as traditional sensitivity and scenario analyses in which a limited set of point evaluations are made. Further, obtaining Monte Carlo estimates with appropriate approximation errors often requires large numbers of samples to be taken, such as $N = 1000$ used in [59].

Therefore, significant research effort is required to develop techniques for reducing the computational cost of decision problem model evaluations, improve the efficiency of Monte Carlo estimation of expected utilities for common problem types, and identify more computationally efficient EUP evaluation strategies, such as the joint sampling technique discussed above.

Further computational difficulties of the VoIA framework arise in the solution of the Prior Decision Problem, as discussed in Section 4.2.v). Due to the non-convexity of Stochastic Optimisation, the determination of the stochastically optimal solution for the Prior Decision Problem may be computationally intractable for many practical decision problem applications. Therefore, the computation of VoI metrics involving y^* may be infeasible for many problems, and so require the use of alternative, approximate metrics.

C. Proof of Non-negativity of Value of Information

For a risk neutral decision maker their expected utility of an outcome is equal to the expected value of that outcome, i.e. they are indifferent between outcomes of equal expected utility regardless of their associated risks. For the purposes of VoIA, assuming a risk neutral decision maker allows outcome value and outcome utility to be used interchangeably.

The non-negativity of the Value of Information metrics presented in Section 3.1 follows directly from the definition of the $\max$ operator and the following two inequalities, which result from the linearity of the expectation operator,

$$\max_a \mathbb{E}_\theta \left\{f(a,\theta)\right\} \leq \mathbb{E}_\theta \left\{\max_a f(a,\theta)\right\} \tag{35}$$

$$\mathbb{E}_\theta \left\{\max_a f(a,\theta)\right\} \leq \max_{a,\theta} f(a,\theta) \tag{36}$$

Starting with the expected utility of the Expected Value Problem (EVP),

$$\hat{y} = \mathbb{E}_\theta \left\{ u\left(\operatorname*{argmax}_{a\in\mathcal{A}} u(a,\bar{\theta}), \theta\right)\right\} \leq \max_{a\in\mathcal{A}} \mathbb{E}_\theta \left\{u(a,\theta)\right\} = y^* \tag{37}$$

which is the expected utility of the Prior Decision Process, thus demonstrating non-negativity of the VSS.

Further,

$$y^* = \max_{a\in\mathcal{A}} \mathbb{E}_\theta \left\{u(a,\theta)\right\} = \max_{a\in\mathcal{A}} \mathbb{E}_z \left\{\mathbb{E}_{\theta|z} \left\{u(a,\theta)\right\}\right\} \leq \mathbb{E}_z \left\{\max_{a\in\mathcal{A}} \mathbb{E}_{\theta|z} \left\{u(a,\theta)\right\}\right\} = y^{**} \tag{38}$$

which is the expected utility of the Imperfect Information Pre-Posterior decision process, thus demonstrating non-negativity of the EVII, and by extension the EVPI, which is a special case of the EVII, specifically its upper bound. The second equality follows from the application of Equation (4), and the inequality from Equation (36).

The inequality,

$$y_p^{**} = \mathbb{E}_z \left\{ \max_{a \in \mathcal{A}} u(a, z) \right\} \leq \max_{\theta \in \Theta,\, a \in \mathcal{A}} u(a, \theta) = y^{\max} \tag{39}$$

follows readily from the linearity of the expectation operator and the definition of the $\max$ operator, and demonstrates the non-negativity of the Value of Control (VoC) [85].

Therefore, it has been proven that the defined value of information metrics are all non-negative, and that the expectations given in Table 2 are listed in weakling increasing order, as shown by Figure 4.

D. Analysis of Incorrect Model Distributions and Erroneous Measurements

The generalised VoIA framework presented in Section 3.4.i) allows for the impact of inaccurate knowledge of the distributions of the uncertain parameters of the system to be quantified in terms of the expected utility obtained by the decision maker.

Consider the case in which the decision maker has incorrect knowledge of both the prior distribution over measurements, z, and the conditional distribution of the system parameters, θ, given a measurement, assuming them to be $\pi_z^{\text{err}}(z)$ & $\pi_{\theta|z}^{\text{err}}(\theta|z)$ when the true distributions are $\pi_z(z)$ & $\pi_{\theta|z}(\theta|z)$ respectively.

For the purpose of a BDA the decision maker determines what they consider to be the stochastically optimal action function, $a^{\text{err}}(z)$ and the corresponding expected utility, y^{err}, by solving the Stochastic Optimisation,

$$a^{\text{err}}(z) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ \mathbb{E}_{\theta|z \sim \pi_{\theta|z}^{\text{err}}(\theta|z)} \left\{ u(a, \theta) \right\} \tag{40}$$

and computing,

$$y^{\text{err}} = \mathbb{E}_{z \sim \pi_z^{\text{err}}(z)} \left\{ \max_{a \in \mathcal{A}} \mathbb{E}_{\theta|z \sim \pi_{\theta|z}^{\text{err}}(\theta|z)} \left\{ u(a, \theta) \right\} \right\} \tag{41}$$

assume for simplicity only one measurement action is available.

However, this action function will be weakly sub-optimal, and will obtain true expected utility,

$$\begin{aligned} y^{\text{true}} &= \mathbb{E}_{z\sim\pi_z(z)}\left\{\mathbb{E}_{\theta|z\sim\pi_{\theta|z}(\theta|z)}\left\{u(a^{\text{err}}(z),\theta)\right\}\right\} \\ &\leq \mathbb{E}_{z\sim\pi_z(z)}\left\{\max_{a\in\mathcal{A}}\mathbb{E}_{\theta|z\sim\pi_{\theta|z}(\theta|z)}\left\{u(a,\theta)\right\}\right\} \\ &= y^{**} \end{aligned} \tag{42}$$

This incorrect knowledge of the system parameter distributions therefore results in a reduction in the obtained expected utility of the decision maker of $\Delta^{\text{err}} = y^{**} - y^{\text{true}}$. This metric can be used to quantify the importance of this distributional error on the results of the VoIA conducted, and hence how sensitive the analysis is to the assumptions surrounding model distributions.

The effect of erroneous measurements can also be studied using this method, as measurement error can be incorporated into incorrect knowledge of the system measurement model, $f(z|\theta)$. For instance, if the measurements are biased, i.e. $z' = z + \delta$, then the erroneously assumed measurement model is given by,

$$f^{\text{err}}(z|\theta) = f(z+\delta|\theta) \tag{43}$$

E. Extensions to VoIA Analysis of LP Based Decision Problems

E.1 Extension of LP-based VoIA to Action & Constraint Violation Cost Uncertainty

E.1.i) Uncertain Control Actions

Consider the case in which the action taken by a decision maker is uncertain, and so when it is attempted to take an action x, the true action implemented is $x + \gamma$ where γ is a random variable. This leads to the underlying LP of the system being,

$$\begin{aligned} \max_{x} \quad & c^T(x+\gamma) \\ \text{s.t.} \quad & A(x+\gamma) \leq b \end{aligned} \tag{44}$$

which is equivalent to,

$$\begin{aligned} \max_{x} \quad & c^T x + c^T \gamma \\ \text{s.t.} \quad & Ax \leq b - A\gamma \end{aligned} \tag{45}$$

the objective term $c^T\gamma$ is independent of the action taken, and so can be neglected from the optimisation, and the uncertain constraint bound can be redefined as $b' = b - A\gamma$, and the corresponding probability density function determined. Hence, this action uncertainty can be re-expressed in standard form and so handled using the framework through the adjusted LP formulation.

More complex action uncertainties, such as $x \to x + \Delta x$, can be incorporated into the framework in a similar manner, with,

$$\begin{aligned} \max_{x} \quad & c^T (x + \Delta x) \\ \text{s.t.} \quad & A(x + \Delta x) \leq b \end{aligned} \tag{46}$$

becoming,

$$\begin{aligned} \min_{x} \quad & c^T (I + \Delta) x \\ \text{s.t.} \quad & A(I + \Delta) x \leq b \end{aligned} \tag{47}$$

The VoI computed when considering such uncertainties could be used to inform decision makers about the optimal control action precision for a given application through the analysis of the trade-off between expected system utility performance and control precision cost.

E.1.ii) Uncertain Constraint Violation Penalisation

Incorporation of an uncertain constraint violation penalisation function $f(Ax - b, \phi)$, where ϕ is a random variable which parameterises the uncertain output of the cost function, is achieved simply by extending the expectation taken of the utility function to include an expectation over the uncertain function parameters, ϕ,

$$\mathbb{E}\{u(x, \theta)\} = \mathbb{E}_{c,A,b,\phi} \left\{ c^T x - f\left(\max[Ax - b, 0], \phi\right) \right\} \tag{48}$$

E.2 Two-Stage Linear Programming Based Decision Problems

Consider a linear system with a two-stage decision structure, where the decision variable x is composed of two vectors, α & β, representing the first and second stage decision variables respectively.

In the EVP the first stage decision seeks to optimise the linear system over both the first and second stage decision variables using mean estimates of the system parameter values. Hence, the first stage EVP is given by the following LP,

$$\begin{aligned} \max_{\alpha,\beta} \quad & \hat{c}^T \left(\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right) \\ \text{s.t.} \quad & \hat{A} \left(\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right) \leq \hat{b} \end{aligned} \tag{49}$$

which has solution, (α_1, β_1).

The first stage decision, α_1, is then enacted on the system. After which, the true parameter values of the system are observed, $(c, A, b) \sim \pi(c, A, b)$. Subsequently, the second-stage decision seeks to optimise the linear system subject to the already implemented first-stage decision, α_1, and the realised system parameter values. Therefore, the second-stage EVP decision could naively be formulated as,

$$\begin{aligned} \max_{\beta} \quad & c^T \left(\begin{smallmatrix}\alpha_1\\ \beta\end{smallmatrix}\right) \\ \text{s.t.} \quad & A \left(\begin{smallmatrix}\alpha_1\\ \beta\end{smallmatrix}\right) \leq b \end{aligned} \tag{50}$$

However, as the first-stage decision was taken assuming the expected value constraint set, which differs from the true constraint set of the system, the implemented first-stage action may result in Equation (50) being ill-defined, leading to there being no feasible second-stage solution. Therefore, the second-stage decision LP must be relaxed to account for the potential for constraint violation through the introduction of a constraint violation parameter γ.

The second-stage LP is therefore given by,

$$\begin{aligned} \max_{\beta} \quad & c^T \left(\begin{smallmatrix}\alpha_1\\ \beta\end{smallmatrix}\right) + f(\gamma) \\ \text{s.t.} \quad & A \left(\begin{smallmatrix}\alpha_1\\ \beta\end{smallmatrix}\right) \leq b + \gamma \\ & \gamma \geq 0 \end{aligned} \tag{51}$$

where $f(\cdot)$ is the constraint violation cost function. Only for linear constraint violation penalisation, $f(\gamma) = v^T\gamma$, does the second-stage decision problem retain its linearity.

Equation (51) is solved, with solution denoted β_2, to yield the determined solution to the EVP, which obtains a utility $u(\alpha_1, \beta_2, \theta)$.

The structure of the two-stage EVP is visualised in Figure 9.

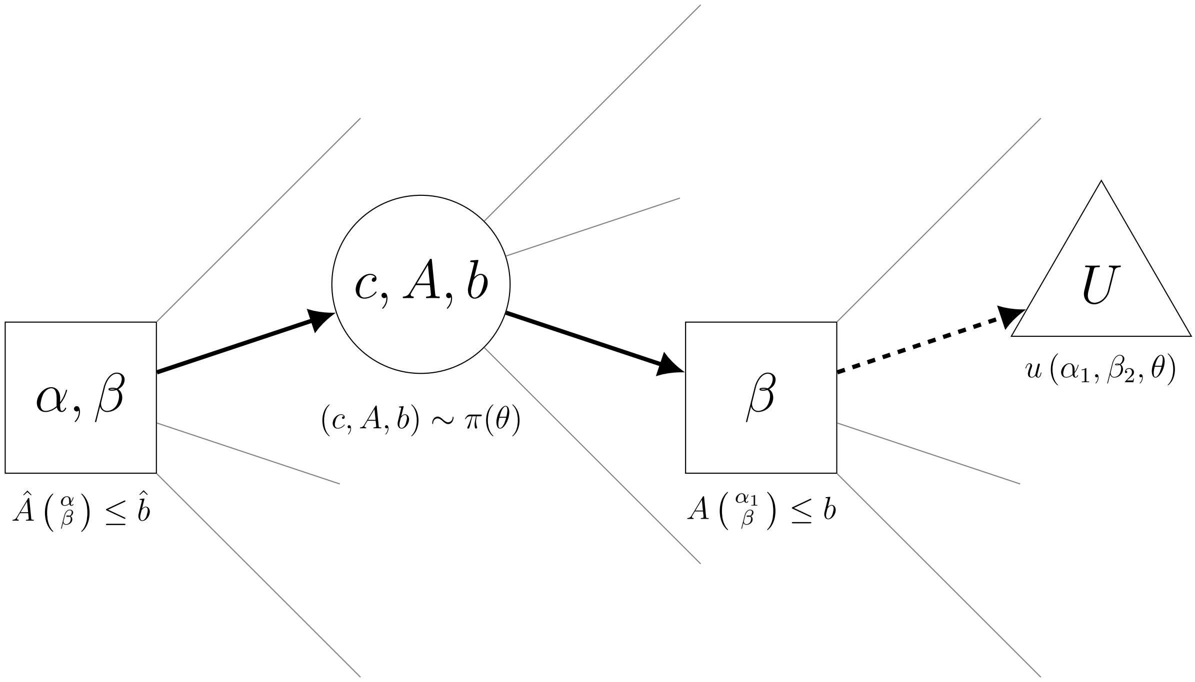

Figure 9: Decision tree representation of two-stage LP-based decision problem

Adapted from Figure 1 of [59]

The expected utility obtained by the EVP solution is determined using Monte Carlo approximation as before,

$$\hat{y} = \mathbb{E}_{c,A,b}\left\{u(\alpha_1, \beta_2, \theta)\right\} \approx \frac{1}{N}\left(\left(\sum_j c^{(j)}\right)^T \binom{\alpha_1}{\beta_2} - \sum_j f\left(\max\left[A^{(j)}\binom{\alpha_1}{\beta_2} - b^{(j)}, 0\right]\right)\right) \quad (52)$$

noting that β_2 depends on the parameter sample.

When computing the expected utility of the Perfect Information Pre-Posterior decision process it is noted that the decision problem with Perfect Information collapses to a one-stage problem. Hence y_p^{**} is computed in an identical manner to the one-stage problem, i.e. via Equation (21) & Equation (22).

Further, the stochastically optimal Prior decision can be determined in a one-stage fashion, with global optimisation performed over both first and second-stage decision variables simultaneously.

Therefore, it has been shown that the LP-based decision problem uncertainty analysis methodology extends readily to two-stage problems, with only the solution strategy of the EVP needing to be adjusted[16].

Within the ESOM context, the first-stage decision variables, α, correspond to the energy system design decision variables, such as the generation asset portfolio, transmission infrastructure, and auxiliary supporting energy infrastructure. Whereas the second-stage decision variables, β, correspond to operational decision variables, such as power/energy transport flows, storage power in/outflows, demand-side response volumes, and generation curtailment volumes.

It can be seen from Figure 9 that the two-stage formulation of the LP-based decision problem is not a true two-stage stochastic decision problem as there is no second uncertainty realisation after the second-stage decision. This formulation decision is made to provide correspondence between the LP-based decision problem and the perfect operational foresight formulation of current state-of-the-art ESOMs [20–22]. The methodology can be readily extended to incorporate an additional uncertainty node, with the second-stage decision adjusted to use expected parameter values as in the first-stage decision. Additionally, the methodology can be further extended to model arbitrary n-stage LP-based decision problems, however motivating applications for such complex Decision System setups are yet to identified.

E.3 Knapsack Problem Illustrative Example

Application of the proposed LP-based decision problem VoI analysis methodology to a stochastic version of the classic textbook problem called 'The Knapsack Problem', as discussed in References [118] & [119], provides useful intuition and insight into the calculations and reasoning behind the method. Visualisation of the EVP feasible region and solution, alongside those for samples from the uncertain parameter distributions, as performed illustratively in Figure 6, illuminates the concept of solution constraint violations resulting from uncertainty in the true constraint set. This tutorial application is left as an exercise for the reader.

[16] Note, the EVPIIP computation strategy would also require an adjusted formulation for the two-stage problem, however this is unlikely to be a desirable analysis to perform as the computation of y^{**} suffers from similar issues of non-convexity and resulting computational cost as the determination of y^*